Automated modelling procedure for acoustic Wave Based Technique models

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Introduction

In vehicle acoustics, the Finite Element Method (FEM) and the Statistical Energy Analysis (SEA) are widely used simulation methods. Yet, for standard vehicle frequency ranges and dimensions both methods suffer from strong limitations. Solving acoustical problems with FEM for higher frequencies $f > 250$Hz, like any element based method, causes high computational costs, while SEA models show poor spectral resolution and no phase information.
In order to overcome these difficulties, a new method, the so-called Wave Based Technique has been established.

Wave Based Technique

The *Wave Based Method (WBM)* or *Wave Based Technique (WBT)* was introduced by Wim Desmet [3] in 1998. The following section provides a short introduction to the WBT for uncoupled acoustic problems. The steady-state sound pressure field for uncoupled acoustic problems is governed by the *Helmholtz equation*

$$\Delta p + k^2 p = -i\rho_0\omega q \qquad \left[\frac{\text{kg}}{\text{s}^2\text{m}^3}\right] \qquad (1)$$

with $k = \frac{\omega}{c}$ the acoustic wave number of the longitudinal waves in the fluid, ω the circular frequency, c the speed of sound, ρ_0 the ambient fluid density, $q = q(x, y, z)$ the distribution of the acoustically applied forces and $p = p(x, y, z)$ the sound pressure distribution. Let $\Omega_p \cup \Omega_v \cup \Omega_Z = \Omega$ be a disjoint partition of the boundary surface of G. Together with the boundary conditions (BCs)

$$p = \bar{p} \qquad [\text{Pa}] \qquad (2)$$

$$\frac{i}{\rho\omega}\frac{\partial p}{\partial n} = \bar{v}_n \qquad \left[\frac{\text{m}}{\text{s}}\right] \qquad (3)$$

$$p = \frac{i}{\rho\omega}\bar{Z}_n\frac{\partial p}{\partial n} \qquad [\text{Pa}] \qquad (4)$$

prescribing pressure $\bar{p}$, normal velocity $\bar{v}_n$ and normal impedance $\bar{Z}_n$ on the boundary sets Ω_p, Ω_v and Ω_Z respectively, the sound field is uniquely defined.
The WBT approximates the solution of the boundary value problem by means of a finite sum:

$$\hat{p}(x, y, z) = \sum_{i=1}^{N} p_i\ \Phi_i + p_q, \qquad [\text{Pa}] \qquad (5)$$

where Φ_i are linearly independent solutions of the homogenuous Helmholtz equation, p_i are the wave function coefficients (to be determined) and p_q solves the inhomogenuous equation.
Via an integral formulation of the BCs, the finite sum is enforced to approximate the boundary values. Of course, the deviation from the solution depends, amongst other criterions, on the actual choice of N, the number of wave functions. For the WBT simulations in this article, the number of wave functions for each direction is given by $N = 2\cdot\max\left(3, \left\lceil\frac{2fL}{c}\right\rceil + 1\right)$, where L is the dimension of a bounding box in this direction sourrounding the problem domain and f the frequency in Hertz.
However, *convexity* of the problem domain is a (sufficient) condition for the convergence of the approximation (5). In case of a non-convex domain, it has to be decomposed into convex sub-domains. Coupling conditions between neighbouring sub-domains ensure pressure and normal velocity continuity. Again, an integral formulation of the BCs and coupling conditions leads to a linear equation system with the wave function coefficients p_i to be the unknown.

WBT implementation and input models

All simulation results in this paper are carried out with the WBT FORTRAN implementation of Achim Hepberger [4] which was developed in a joint project with the Acoustic Competence Centre Graz (ACC). As the input models for this implementation play an important role further in this article, its attributes are now introduced in detail: The model consists of pairwise disjoint and convex polyhedra. The polyhedra's surfaces

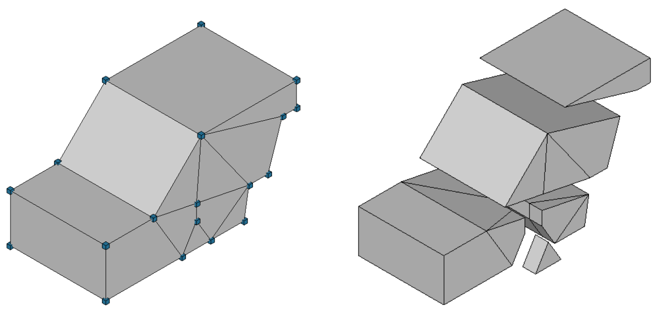

Figure 1: A WBT input model consists of pairwise disjoint, convex polyhedra. Its surface polygons are triangles and plane rectangles.

are triangles or plane rectangles. This set of faces consists of the outer boundary faces and the interior, i.e. coupling faces. To each of the outer faces a BC is prescribed; to the coupling faces the corresponding coupling conditions are applied.

WBT model generation

The WBT model generation is part of MAGNA STEYR's standard acoustic modelling process [1].

Acoustic trimmed body FEM models are generated out of CAD (Computer Aided Design) models. FEM models are often used as basis for SEA models. Because of their coarseness, SEA models again (more precisely: their geometric representation in common software products such as VAOne from ESI Group for example which is used at MAGNA STEYR) are suitable to be the starting point for WBT hull models: The passenger compartment cavity(ies) is extracted and the faces are meshed into plane polygons (Fig. 2). This hull model is the input for

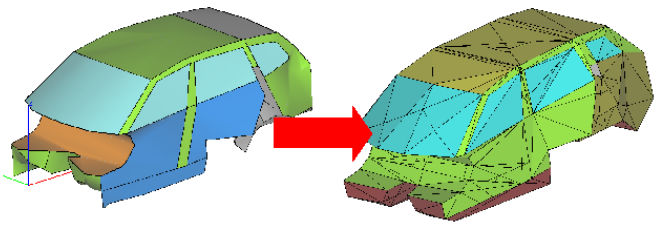

Figure 2: The passenger compartment cavity subsystem of an SEA model is the starting point for the WBT hull model generation. The faces are meshed into plane polygons.

the convex decomposition algorithm which is described later in this article. After the decomposition process, post processing steps clean up the model from tiny or thin faces and solids. The BCs are also defined in and extracted from the SEA model. During decomposition the assignment of the faces to the corresponding BC is preserved. Together with the acoustic excition configuration (e.g. point source or velocity BCs), the frequency domain definition and the material definition of the fluid, the WBT model is fully specified and ready for calculation.

Modelling requirements

The question arises, which requirements the decomposition algorithim must fulfill. The most important are: •Decomposition into *convex* sub-domains. •Preservation of BC assignment. •Flexibility (due to many geometric changes during project). •Fast (manual decomposition $\sim$ 1 month). •Stable simulation results (due to actual decomposition).
The task is now to introduce an algorithm which is able to fulfill these requirements and is integrable into the standard modelling process.
In case of 2D models, WBT performance studies were carried out in the past [5]. The main results are: **1.** The WBT exceeds the FEM in terms of calculation time and quality when the model has a moderate geometric complexity. **2.** The number of convex sub-domains should be kept small in order to gain computational speed. **3.** The convex sub-domains should have similar size and sharp boundary edges should be avoided.
Analoguous investigations for the 3D case lead to similar statements. However, from the geometric point of view, a proper 3D mesh generation is a harder task.

Decompostion algorithm

The task is to sub-divide a given polyhedron into disjoint, convex sub polyhedra with respect to the above stated criterions. The basic idea for convex decomposition comes from B. Chazelle [2]. The idea is to identify and sub-sequently resolve those geometric spots which cause non-convexity. Those spots are called *notches* and are

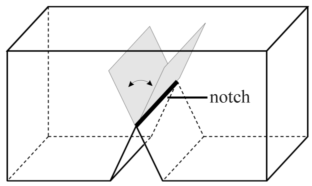

Figure 3: A polyhedron with a reflex angle which causes non-convexity. The corresponding edge is called *notch*.

edges which connect 2 faces with an interior angle $> 180°$ (Fig. 3).
After detection of all reflex angles, plane cuts are subsequently applied through all notches so that these cuts are all within the valid angle domain as indicated in figure 3 (between the two gray faces). This way the reflex angle is resolved.
Let's assume that a few cuts have been carried out and the next one is to choose: There are two decisions to make: **1.** The choice of the notch through which the cut should be carried out and **2.** the choice of the actual angle within the valid angle domain.
A meshing strategy is presented in order to choose an optimal cutting plane with respect to the following criterions:

1. Minimizing the number of notches after cut
2. The two cutted sub-polyhedra should be as balanced as possible, i.e. similar in volume
3. Minimizing the number of faces after cut
4. Taking the longest notch

In general, applying these criterions one by one will lead to different cutting planes. Therefore, by means of a weigthing function, a meshing strategy is created out of these sub-strategies. Such a weighting function may look like as follows: Let a_1, a_2, a_3, a_4 be the weights of the four above mentioned criterions, $a_i \geq 0$. $a_i > a_j$ means that criterion i is more important than j. Let further be x_1, x_2, x_3, x_4, $0 \leq x_i \leq 1$, the corresponding assessment values, where $x_i = 0$ means that criterion i is not or badly fulfilled and $x_i = 1$ indicates, that criterion i is fulfilled or best fulfilled (compared to other cuts). Then the (weighting) function $F = \sum_{i=1}^{4} a_i x_i$ defines an overall cut strategy consisting of four sub-strategies.
Let now be given a non convex polyhedron. Each possible cut is assessed by means of the above mentioned function. The cut with the highest value is carried out. The same procedure is then applied to the 2 sub-polyhedra and so on until all sub-polyhedra are convex. It is clear that different weighting values will generally result in different decomposed models.
Figure 4 illustrates the decomposition procedure. Even though the number of notches may increase during the decomposition process when notches are cut into 2 pieces by other cutting planes, it can be shown [2], that the algorithm terminates after a finite number of steps.

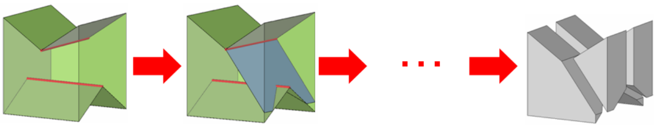

Figure 4: The decomposition algorithm identifies all notches (left figure). After that a cutting plane sub-divides the polyhedron (blue face in middle figure) and so on until all sub-polyhedra are convex (right figure).

WBT Simulation

The preceding section showed that different weighting strategies lead to different WBT input models. It arises the question which impact these different decompositions have on the WBT simulation result. Therefore simulations with a representative model are investigated.
Let's consider a standard passenger compartment (Fig. 5). A normal velocity BC ($v_n = 0.001$m/s), exciting

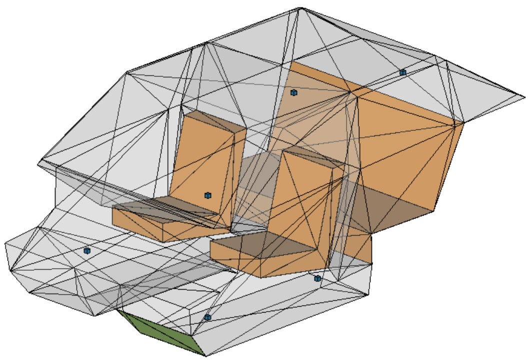

Figure 5: A simplified standard passenger compartment model with dimensions (in [m]) ($l \times w \times h = 2.8 \times 1.5 \times 1.2$). The sound pressure field is excited by a normal velocity BC in the front lower left part (green face). The brown faces indicate absorbing BCs, all other faces are considered sound hard. The 6 uniformly distributed points in the interior show the response locations.

the sound field, along with absorbing ($Z_n = 1.97e3 - i\ 3.35e3$Ns/m^3) and sound hard BCs are applied to the model. The frequency range is 1 - 500Hz, the calculations are carried out in 1Hz steps. The results are compared at 6 uniformly distributed points in the interior.

WBT model variants

The task is to compare different WBT model variants with respect to simulation quality as well as simulation and modelling time. Amongst 9 automatically decomposed WBT models also 4 manual WBT models and a FEM reference model are compared. The 9 automated models are generated by varying the meshing strategy by different choices of the weighting coefficients $[a_1, a_2, a_3, a_4]$ defined in the preceding section. 2 automated models are shown in figure 6. It can be seen, that there are big differences in terms of geometric model parameters between the different models. For example, the ratio of the number of resulting convex sub-domains between the 2 models is greater than 2.
The two manually decomposed models are illustrated in figure 7. The left one is decomposed with the target of minimizing the number of sub-domains. This leads to

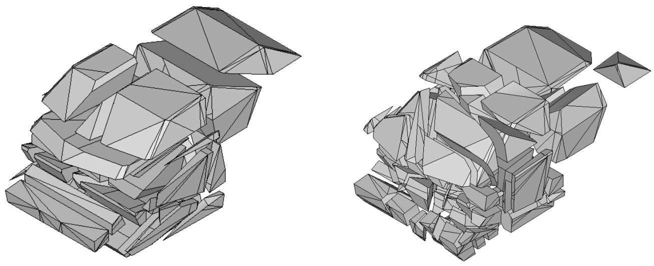

Figure 6: Two automatically decomposed WBT models with the meshing strategies $[a_1, a_2, a_3, a_4] = [1, 0.2, 0.1, 0.1]$ (left) and $[0, 1, 0, 0]$ (right; see the preceding section for more details). The left model consists of 38 convex sub-domains, while the right one has 83 solids.

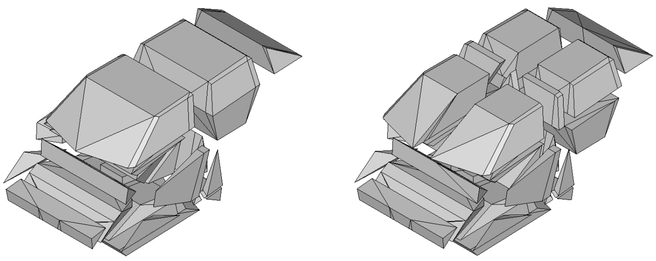

Figure 7: Two manually decomposed WBT models. The left one *(man1)* has 27 solids, the right one *(man2)* 37. Both models are additionally calculated with an increased number of wave functions.

highly unbalanced volume ratios between several solids. Therefore a second manual model is generated (right), in which such ratios are avoided by simply cutting the larger sub-domains in 2 parts.
In addition, a FEM simulation with Comsol Multiphysics is carried out for reference purpose. The model is meshed with tetrahedral Lagrange elements 2$^{\text{nd}}$ order and consists of about 29.000 degrees of freedom. The model is directly solved for each frequency step.

Results

The figures 8 and 9 present the calculated sound pressure levels at the 6 response points. Up to

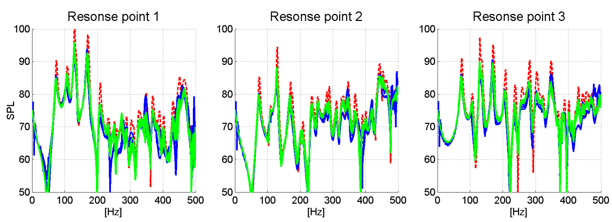

Figure 8: WBT Simulation result: Sound pressure level (SPL) at response points 1, 2 and 3. Dotted red line: FEM, blue: manually decomposed models, green: automatically decomposed models.

250Hz, an agreement better than ±1dB (except peak frequencies) is observed. For frequencies > 250Hz, there are deviations up to 5dB for a few frequency ranges. However, it should be noted that the largest deviations occur between the FEM solution and the manually decomposed models. The automatically decomposed models have generally less deviation to the FEM solution, as is shown in table 1. This is mainly

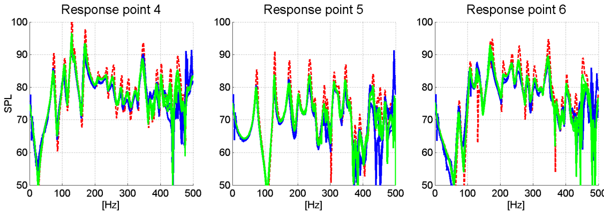

Figure 9: WBT Simulation result at response points 4, 5 and 6.

due to the following reasons which was studied by Van Hal [5]: manual decomposed models tend to have more faces where boundary *and* coupling faces lie in the same plane. This is the case when cutting planes are used in which boundary faces are lying (Fig. 10). As this leads

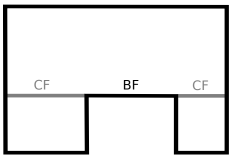

Figure 10: When boundary faces (BF) lie in the same plane as coupling faces (CF), the wave functions have to approximate non continuous values along this plane.

to non continuous values along such a plane, the wave functions don't approximate these values well.
Furthermore, among the automated model results, the deviation is much smaller compared to the deviation to FEM and manual result. This indicates, that the

Model	Mean dev	Model	Mean dev
[0|1|0|0]	1.77	[0|0|0|1]	2.38
[0|0|1|0]	1.95	[1|0|0|1]	2.39
[1|0|1|1]	2.12	ma2 WF+	2.75
[1|0.2|0.1|0.1]	2.13	ma1 WF+	2.88
[1|0.5|0.5|0.5]	2.18	man2	3.23
[1|1|1|1]	2.20	man1	3.43
[1|0|0|0]	2.27		

Table 1: Model related deviation of all WBT models from the FEM result. Automatically decomposed models have smaller deviation from the FE result. "WF+" means an increase of wave function by 1 for each direction.

automated models generate stable results.
Figure 11 illustrates the frequency dependency of the deviation to the FE result. The deviation clearly increases with frequency. This is mainly due to the fact that for higher frequencies, there are more modes per bandwidth. As mentioned above, the deviation is greatest around such peaks.

Conclusions

The simulation results of the autmatically decomposed models have little variation. Compared to manual WBT models, they show even better correlation.
From the modelling time and flexibility point of view, the automated models have big advantages compared to manual decomposition. Especially with large models

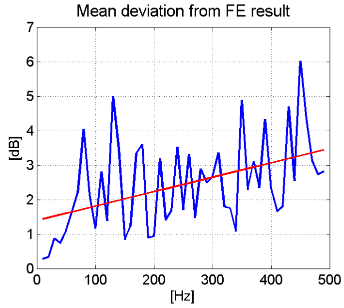

Figure 11: Blue curve: Mean deviation of the WBT results to the FE result versus frequency. Red line: Approximating linear function. Before calculating the deviation, all results were constant-bandwith averaged (Bandwidth: 10Hz). The deviation increases with frequency.

(many faces and notches) the algorithm proves to be efficient.
When it comes to simulation time, the manual models have advantages because of their smaller number of solids. However, because of the difficulties mentioned above, the number of wave function has to be increased in most cases. By doing so, however, the simulation time increases. Experience shows, that in this case, the simulation time of manual models is more or less equal to that of smaller automatically decomposed models.
In summary it can be said, that the introduced WBT modelling algorithm, which is part of MAGNA STEYR's standard modelling process, proves to be efficient for passenger compartment models compared to manual modelling techniques. The modelling times are substantially smaller and the quality is generally higher.

References

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