Resonance frequencies and sound radiation of musical woodwind instruments

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Introduction

Decoupling the acoustical resonator from the excitation mechanism allows to analyze the influence of the resonators geometry on the acoustical behavior of the whole woodwind instrument. Such investigations are done typically in the frequency domain. The aim of our approach is to provide a tool which acts in the time domain to overcome some limitations of frequency based approaches. Numerical investigations in the time domain offer some advantages. Acoustical waves can be tracked directly, which gives a better insight how connections, tone holes and bore perturbations may influence the wave propagation and act as sound sources. In contrast to methods formulated in the frequency domain effects based on the superimposed and non-homogeneous mean flow and also transient effects can be studied.

The impulse reflectometry is a common approach to investigate acoustical systems experimentally [1, 2]. As an alternative approach we apply this method numerically. Such investigations need to track acoustical waves over a long time period. We use a high-order discontinuous Galerkin formulation to achieve the required accuracy and solve the unsteady, non-linear Euler equations [4].

The aim of this work is to demonstrate the general practicability of this approach. As a test configuration we studied the resonance frequencies and radiation pattern of the bassoon.

Configuration

The investigations base on a two-dimensional model of a bassoon. The two-dimensional approach causes some limitations:

1. The sound radiation differs from the real radiation behavior.
2. The instruments body and especially the crook form an unrealistic obstacle for acoustical waves.
3. Also the geometry adaption to a two-dimensional model causes geometry modifications.

In spite of these limitations the reduction to two dimensions offers an efficient computation of the acoustical resonator with a moderate numerical effort. Contrary to a full three-dimensional calculation this make the study of a complete set of geometric modifications including the whole fingering possible. This is necessary to modify the instruments geometry systematically.

In Fig. 1 a detail of the test geometry and the numerical grid is shown. The outer domain forms a circle with the diameter of 1 m around the instruments centroid, which is bordered by absorbing boundary conditions [4]. The grid size varies between 1 mm in an around the instrument and 10 mm in the far field. With this resolution acoustical waves up to 15 kHz can be tracked. The grid consists of 11,602 elements with the polynomial degree of 3. The polynomial basis determines a discretization within an element. This resulting sub grid is not shown here. An explicit TVD Runge-Kutta time integration scheme is applied to conserve the accuracy of the numerical method. Depending on the numerical grid the time step size is $2 \cdot 10^{-7}$ s and the sampling rate 5 MHz respectively.

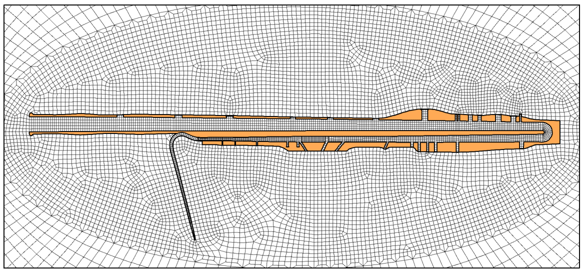


Figure 1: Geometry and numerical grid.

As input impulse we defined a half sine wave with a length of 5 cm and a pressure amplitude of approximately 7 Pa. This provokes uniformly frequencies up to 10 kHz. The critical frequency is about 14 kHz.

Numerical results

Impulse response

In Fig. 2 the time history of the traveling impulse is illustrated. The impulse starts at the wings entryway (see Fig. 2 (a)), radiates at the open tone holes and reflects at cross section perturbation (Fig. 2 (b–d)). In contrast to the playing situation the acoustical waves can leave the instruments body at the entryway unaffected.

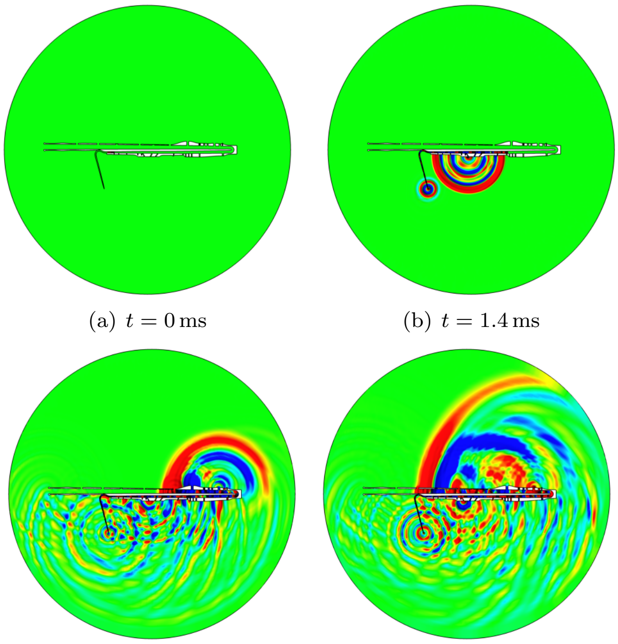
Fig. 3 shows the static pressure fluctuation inside the wings entryway over the first 5 ms. This demonstrates how bore perturbations affect the traveling impulse.

Complex reflection function

The complex reflection function is defined as the quotient of the pressure answer $\boldsymbol{p}(f)$ to the input signal $\boldsymbol{p}_{ref}(f)$ following

$$|\boldsymbol{r}(f)| = \frac{|\boldsymbol{p}(f)|}{|\boldsymbol{p}_{ref}(f)|}\,. \tag{1}$$

Fig. 4 gives a comparison of the reflection function of



(a) $t = 0\,\mathrm{ms}$ (b) $t = 1.4\,\mathrm{ms}$ (c) $t = 4.0\,\mathrm{ms}$ (d) $t = 5.2\,\mathrm{ms}$

Figure 2: Pressure field at different time steps; static pressure fluctuation $p' = -50 \ldots 50\,\mathrm{ms}$; note G3 (197.3 Hz).

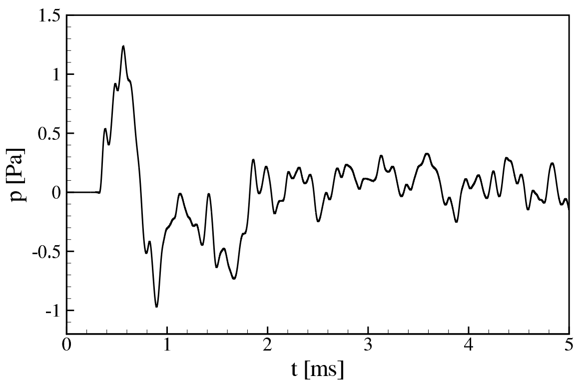


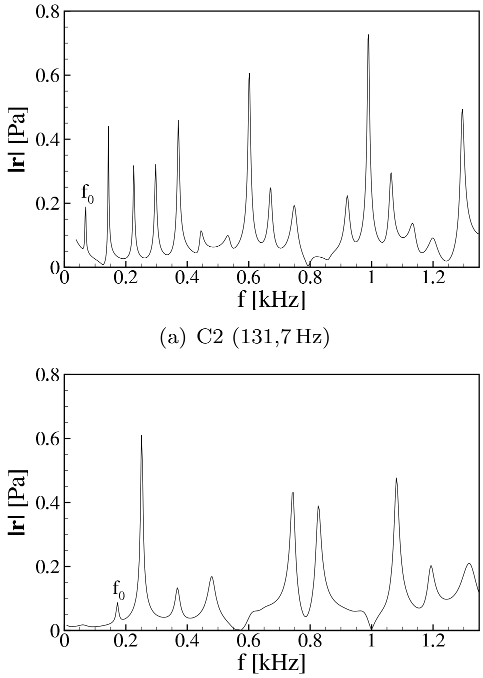
Figure 3: Time history of the pressure fluctuation inside the wing; note G3 (197.3 Hz).

the fingering C2 and G3, which are estimated at the instruments entryway. The reflection function shows the fundamental frequency and also the first overtones. As typically for the bassoon the amplitude of the fundamental frequency f_0 is below the first overtone.

In Tab. 1 the corresponding fundamental frequencies of different notes are compared with the frequencies known by literature. The correlation of the first two notes (B flat 1 and C1) is in the expected range. For the other notes the estimated frequencies differ from the expected values, according to the two-dimensional model.

Sound radiation

It is known from literature that the sound radiated by the bassoon features a strong directional characteristic [3]. In Fig. 5 the sound radiation pattern for the fundamental frequency and the first three overtones of the note C2 is given. The numerical simulations show a similar behavior compared to the literature: Every oscillating mode radiates with its own distinct characteristic. These



(a) C2 (131,7 Hz)

(b) G3 (394,7 Hz)

Figure 4: Reflection function of the Note C2 (131,7 Hz) and G3 (394,7 Hz), estimated at the instruments entryway.

Note	f_{ref} [Hz]	f [Hz]	Δf [Cent]
B flat 1	58,7	58,9	6
C1	65,9	68,3	62
G1	98,7	108,0	156
C2	131,7	164,8	388
G2	197,3	173,0	-228
C3	263,4	277,6	91
G3	394,7	311,0	-413
C4	526,8	504,5	-75

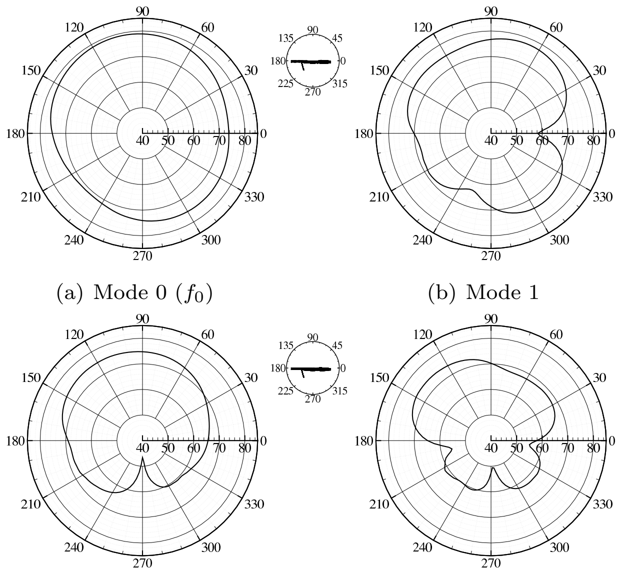
Table 1: Numerical estimated fundamental frequencies, Notes B flat 1 (58,7 Hz) to C4 (526,8 Hz).

characteristics vary also with the played note. As a consequence the comprehensive study of the sound radiation requires:

1. the investigation of a multitude of notes and
2. the calculation of a large number of overtones for every note.

Conclusion

We presented a method to study the acoustical behavior of woodwind instruments by performing the widely-used impulse reflectometry numerically. For this we solved the non-linear, compressible and unsteady Euler equations. To achieve the numerical accuracy which is required to track acoustical waves over a long time period we implemented a high-order discontinuous Galerkin formulation. The time domain based approach overcomes some limitations of frequency based methods. Waves can be tracked directly which gives a better insight in the resonators behavior and transient phenomena and also a



(a) Mode 0 (f_0)

(b) Mode 1

(c) Mode 2

(d) Mode 3

Figure 5: Sound radiation of different modes of the note C2.

non-homogeneous mean flow can be investigated.

To demonstrate the general practicability of this approach we investigated the resonance frequencies and radiation pattern of a two-dimensional model of a bassoon. According to the two-dimensional approach the resonance frequencies differ from the expected values. The numerical studies showed distinct radiation pattern which varies substantially with the played note and also between the fundamental frequency and the higher modes. This radiation behavior is well known from the literature.

References

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