Modal Analysis of a Fluid inside and around a Recorder

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Introduction

The work, presented in this paper, is aiming at a three-dimensional simulation of the sound evolution, sound propagation and sound radiation of a recorder. The main focus of the sound evolution is on a realistic reproduction of the excited field, which mainly depends on the resonator, and on a periodic energy transfer between pressure and velocity field in this area. For the sound spectrum a modal analysis of the recorder fluid is realised. The goal is to study the different parameters as a pre-stage of form optimizations of systems with complex geometries. We want to be able to predict changes in the sound resulting from changes in the geometry (labium, sound holes, inner tube).

The fluid inside and close to the recorder is meshed by Lagrangian tetrahedral finite elements. To obtain results in the far field of the recorder, complex conjugated Astley-Leis infinite elements are used [1, 2]. To apply these infinite elements the finite element domain has to be meshed either in a spherical or in an ellipsoidal shape. Advantages and disadvantages of both shapes regarding the recorder will be shown in the following. Examples of the modal analysis of the fluid for different notes will also be presented.

Acoustic radiation problem

As already mentioned in the introduction, it is possible to use a spherical or an ellipsoidal shape of the finite element domain when adding infinite elements, see Figures 1 and 2. An advantage of a spherical shape is that the mass matrix of the infinite elements is zero. But with

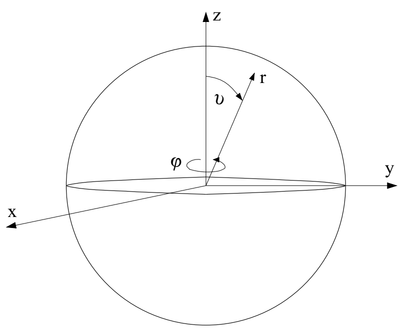

Figure 1: spherical FE domain shape

regard to the recorder, an ellipsoidal domain shape is advisable since the ellipsoid can be placed close around the recorder, keeping the degree of freedom significantly smaller than with a spherical domain.

The boundary value problem for an acoustic radiation problem of an inviscid and incompressible fluid consists of the Helmholtz equation (1), the Neumann boundary con-

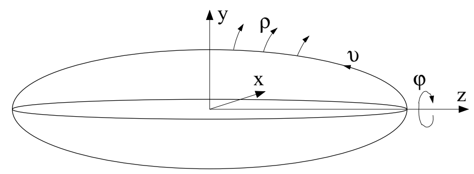

Figure 2: ellipsoidal FE domain shape

dition (2) and the Sommerfeld radiation condition (3),

$$\begin{aligned} -\Delta p(\boldsymbol{x}) - k^2 p(\boldsymbol{x}) &= 0 \qquad \text{with} \quad \boldsymbol{x} \in \Omega & (1)\\ \frac{\partial p(\boldsymbol{x})}{\partial \boldsymbol{n}} &= \tilde{g}(\boldsymbol{x}) \quad \text{with} \quad \boldsymbol{x} \in \Gamma & (2)\\ R\left\{\frac{\partial p}{\partial R} - ikp\right\} &\to 0 \qquad \text{for} \quad R \to \infty\,. & (3)\end{aligned}$$

The matrix formulation of the problem can be written as

$$(\boldsymbol{K} - ik\boldsymbol{D} - k^2\boldsymbol{M})\boldsymbol{p} = \boldsymbol{b}\,, \tag{4}$$

with $\boldsymbol{K}$ being the stiffness, $\boldsymbol{D}$ the damping and $\boldsymbol{M}$ the mass matrix; $\boldsymbol{p}$ contains the pressure in each node and $\boldsymbol{b}$ is the right-hand-side vector which can contain all kinds of forces acting on the system.

For the modal analysis, Equation (4) is transformed into the following state space formulation

$$\begin{bmatrix} \boldsymbol{\Phi}_l \\ \boldsymbol{\Psi}_l \end{bmatrix} \left(\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{K} \end{bmatrix} - \lambda \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{M} & \boldsymbol{D} \end{bmatrix} \right) \begin{bmatrix} \boldsymbol{\Phi}_r \\ \boldsymbol{\Psi}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \tag{5}$$

with $\boldsymbol{\Phi} = \lambda\boldsymbol{\Psi}$, $\lambda = -ik$ and the identity $\boldsymbol{I}$.

Modal analysis

For the modal analysis a threedimensional finite element model is build in Ansys 11.0, meshing everything inside and around the recorder within an ellipsoidal domain. Figure 3 shows a section of the fluid model, meshed with tetrahedral elements. Herein second order Lagrangian elements are used. In this figure the mesh inside the recorder can be seen, as well as the not-meshed part of the recorder itself and part of the outer mesh. It is known that the eigenvalues of an unbounded domain are complex conjugated and a great deal of them cluster close to the imaginary axis. There are also eigenvalues which lie along lines, the number of these lines depends on the polynomial order of the infinite elements [5]. If the polynomial order is odd, additional eigenvalues appear along the real axis.

The interior eigenvalues are characterised by the fact that their real part is significantly smaller than their imaginary part. In this work, the focus is on the computation of internal modes, because they coincide

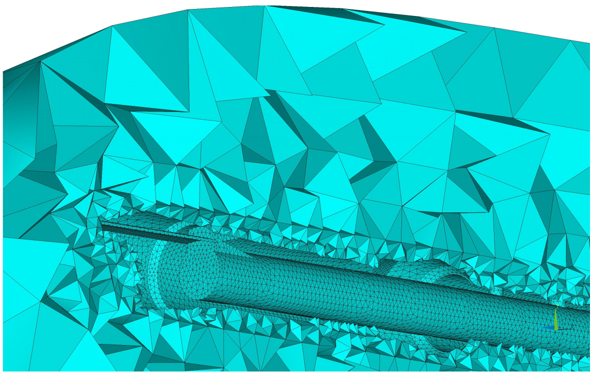

Figure 3: section of FE model of the fluid

with the recorder notes. Due to that, only eigenvalues close to the imaginary axis are computed.

Figure 4 shows the obtained eigenvectors for notes f′ and d″. For note f′ all tone holes are closed and the wavelength equals the length of the inner recorder hole. By contrast, for note d″ the lower five tone holes are open and all others completely closed.

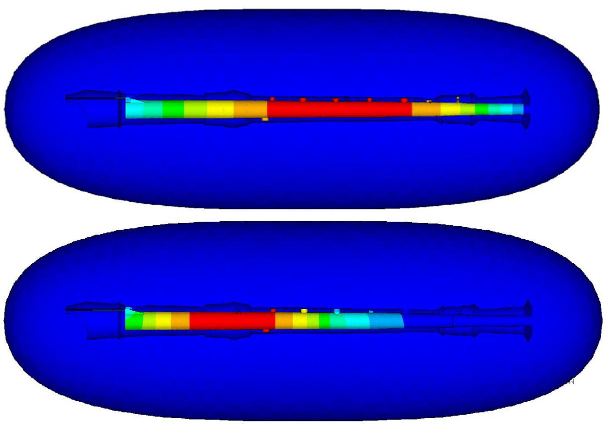

Figure 4: note f′ with all tone holes closed (above) and note d″ with the lowest five tone holes open (below)

In Figure 5 the convergence behaviour of note d″ is shown. Displayed is the frequency over the degree of freedom. According to the "Musical Instrument Digital Interface" table (MIDI-table) and subsequent conversion of the frequencies to the tuning of the recorder being considered in this work, the frequency of d″ should be at 590 Hz. The slight difference between the required and the obtained frequency was expected, since all computations were made without consideration of a volume flow. The frequencies according to the MIDI-table take into account that a musician is actually playing the notes, that is to say that a volume flow is present.

Summary and future prospects

This paper presented the three-dimensional numerical simulation of the sound spectrum and the propagation of the acoustic noise inside and around a recorder. The results of the modal analysis for two different notes were presented. Slight differences appeared between the computed and expected eigenvalues. This was due to

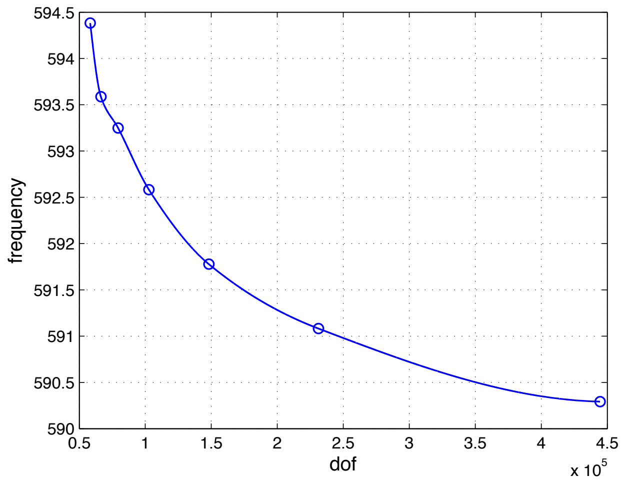

Figure 5: convergence of d″

the fact that the volume flow had not been taken into consideration in these computations. Therefore, one of the future tasks will be the inclusion of the volume flow in the modal analysis. One interesting fact there will be to study the influences of the volume flow and its effect on the sound.

Another future prospect will be the interface to fluid mechanics. In fluid mechanics a Navier-Stokes solver is used to determine the sound forming mechanisms on the labium and the wave propagation inside the recorder. Pressure and velocity as time- and space-dependent functions are obtained. The results can be implemented in the acoustic computations after applying a FFT and an interpolation on the coarser FE mesh. With an identification and positioning of sources the excitation mechanisms for the sound propagation inside and around a recorder can be displayed. With this interface we will be able to determine intonation and sound of the recorder.

Furthermore, the influences of geometric modifications will be studied.

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