Wave Based Prediction Technique for Sound Radiation Analysis

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Introduction

Recently, the wave based prediction technique (WBT) has been developed as an alternative method for solving steady-state acoustic problems in the mid-frequency range. Based on an indirect Trefftz approach [1] the WBT has proven to be a robust prediction tool for interior acoustics [2]. However, a considerable class of real-life acoustic applications involves the analysis of problems in unbounded spatial domains, such as sound scattering and sound radiation problems. Besides the boundary element method, which is based on boundary integral formulation of the governing differential equation, various strategies employing the standard finite element schemes were developed in order to tackle unbounded problems, such as non-reflecting boundary condition, infinite elements or perfectly matched layers. Although based on different approaches, all these concepts have the same basic idea in common, namely introducing an artificial truncation boundary that divides the infinite domain into two regions – the bounded and unbounded part.

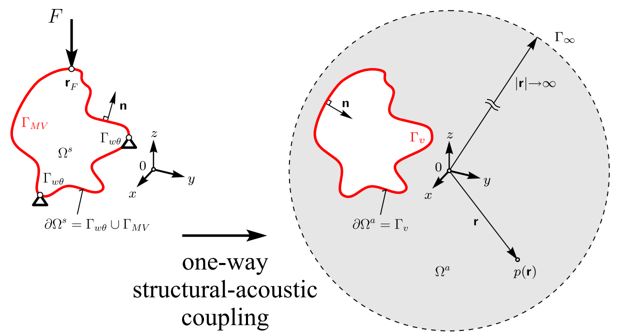
This paper outlines the recent developments of the novel wave based approach and its application for three-dimensional sound radiation analysis under anechoic conditions. By adopting a similar strategy, as discussed above, the model is divided into two parts. The conventional interior formulation is used in the bounded part, while some novel functions, which inherently satisfy the Sommerfeld radiation condition, are adopted in the unbounded part of the wave model. Application to an industry-sized problem demonstrates the practical applicability of the wave based approach.

Problem Definition

Whenever a structure is surrounded by fluid the vibro-acoustic behaviour of the system is influenced by the mutual coupling interaction between the structure and fluid. A specific class of problems, however, allows the strong mutual coupling may be neglected and the both subsystems considered as being uncoupled. In particular from a computational point of view this distinction has an essential importance.

Consider a three-dimensional one-way coupled structural-acoustic unbounded problem, as shown in Figure 1. Since the mutual coupling interaction between the structure and fluid is considered to be very weak, in this particular case, only the one-way structural loading effects on fluid are taken into account. This allows the problem may be decomposed into two parts and solved separately step by step.

The structural part consists of a closed boundary with an imposed free boundary condition Γ_{MV} except for the discrete constraint points corresponding to a clamped type of boundary condition $\Gamma_{w\theta}$, see Figure 1(a). The system is excited by a harmonic point force F applied at the position $\mathbf{r}_F$.



(a) (b)

Figure 1: An one-way coupled structural-acoustic unbounded problem: (a) the structural and (b) acoustic part of the problem.

The acoustic part consists of a closed boundary surrounded by fluid which forms an unbounded acoustic domain Ω^a, see Figure 1(b). Assuming the system is linear, the fluid is inviscid and the process adiabatic, the steady-state pressure response $p(\mathbf{r}, t) = p(\mathbf{r})e^{j\omega t}$ at an arbitrary position $\mathbf{r}$ within the solution domain Ω^a is governed by the homogeneous Helmholtz equation

$$\triangle p(\mathbf{r}) + k^2 p(\mathbf{r}) = 0, \tag{1}$$

where $\triangle \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ represents the Laplace operator in Cartesian coordinates, $\mathbf{r}$ the position vector, t the time, $j = \sqrt{-1}$ the imaginary unit, ω the circular frequency, c the speed of sound and $k = \omega/c$ the wave number. At the boundary of the problem $\partial\Omega^a = \Gamma_v$ the normal velocity boundary condition is imposed

$$\mathbf{r} \in \Gamma_v : \mathcal{L}_v(p(\mathbf{r})) = \frac{j}{\varrho\omega}\frac{\partial p(\mathbf{r})}{\partial n} = \bar{v}_n(\mathbf{r}), \tag{2}$$

with $\frac{\partial}{\partial n}$ the normal derivative and $\bar{v}_n(\mathbf{r})$ the prescribed normal velocity. Moreover, as the solution domain Ω^a is unbounded, an additional Sommerfeld radiation condition has to be imposed at Γ_∞ in order to ensure that no acoustic energy reflections occur at the infinity

$$\mathbf{r} \in \Gamma_\infty : \lim_{|\mathbf{r}| \to \infty} \left[|\mathbf{r}| \left(\frac{\partial p(\mathbf{r})}{\partial |\mathbf{r}|} + jkp(\mathbf{r}) \right) \right] = 0. \qquad (3)$$

The concept of the Wave Based Technique

The WBT [2] is based on indirect Trefftz approach, in which it incorporates an a priori knowledge of the solved problem. The field variables are expressed in terms of globally defined shape functions, which are the exact solutions of the homogeneous governing differential equation (1), but which do not necessarily satisfy the boundary conditions. In the wave based formulation for interior problems, these functions represent evanescent and propagating waves and form the wave function set. Using a weighted residual scheme, the residual errors arising at the boundary are enforced to zero in an integral sense. Solution of the resulting system of algebraic equations yields the contribution factors of the wave functions. The wave models are substantially smaller than equivalent finite element (FE) [5] and boundary element (BE) [6] counterparts and exhibit an increased computational efficiency.

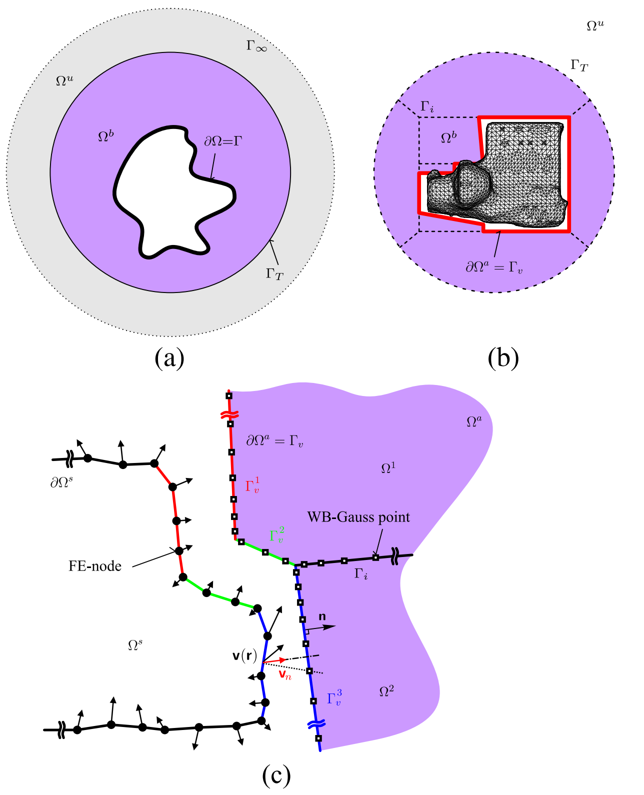


Figure 2: The application of WBT for one-way coupled structural-acoustic unbounded problems: (a) the concept of a truncation boundary Γ_T, (b) the modelling strategy and (c) a generic multi-local-velocity (MLV) algorithm.

In order to tackle the problems involving unbounded domains, an additional treatment of the interior WB formulation is required [4]. By introducing an artificial truncation boundary Γ_T, see Figure 2(a), the solution domain Ω^a is divided into a bounded and unbounded part $\Omega^a = \Omega^b \bigcup \Omega^u$. In the bounded part the WB formulation for interior problems is applied [2, 4], whereas in the unbounded part functions which additionally satisfy the Sommerfeld radiation condition (3) are employed [3].

To couple the structural and acoustic problem which are based on different computational techniques and thus use incompatible numerical models, a generic multi-local-velocity (MLV) algorithm is made use of [7], see Figure 2(c). To each WB boundary part Γ_v^i a user specified region of the FE mesh is assigned. Within this target region a corresponding finite element is sought, where the projection of the GP on to the FE mesh meets some algorithm specific criteria. After the corresponding finite element counterpart is found, the velocity vector $\mathbf{v}(\mathbf{r})$ for the given FE-GP mapping is determined using the nodal velocities of the corresponding finite element by finite element interpolation (shape functions). For WB boundary condition $\bar{v}_n(\mathbf{r})$, only the vector component normal to Γ_v^i is taken.

Validation example

To show the applicability of the proposed approach to industry-sized problems an assembly consisting of a four cylinder car engine block and gearbox with the main dimensions of 0.995 × 0.523 × 0.535 m was considered as a validation example, see Figure 3.

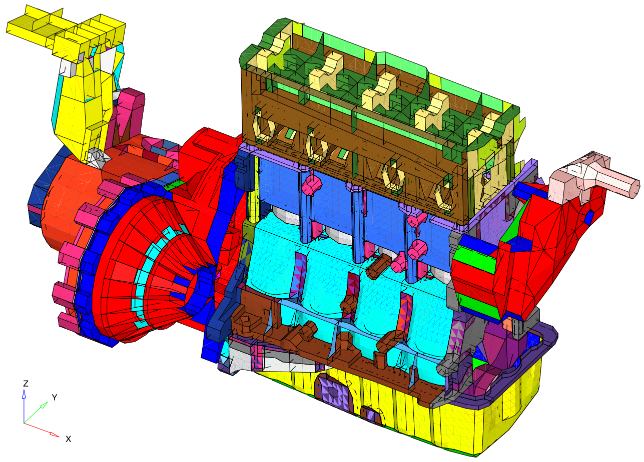


Figure 3: The car engine block-gearbox assembly: an initial FE model.

Due to the intellectual property related aspects it is a common practice in the industrial engineering that the data are exchanged merely on a FE model level, so that these are often the only available data containing the geometrical information. In the most cases, these models, however, arise from a different analysis fields such as structural dynamic or fatigue simulation. Since these calculation procedures pose specific requirements for computational meshes, these do not have to be necessarily well suitable for a sound radiation analysis in which some geometrical features (e.g. small holes and ribs) have a negligible effect and may be omitted.

In this respect, the initial rather complex FE model consisting of shell and volumetric elements was simplified for the validation purposes. The engine suspension was omitted and the original FE model was wrapped by a coarser mesh consisting of linear triangular shell elements, see Figure 4.

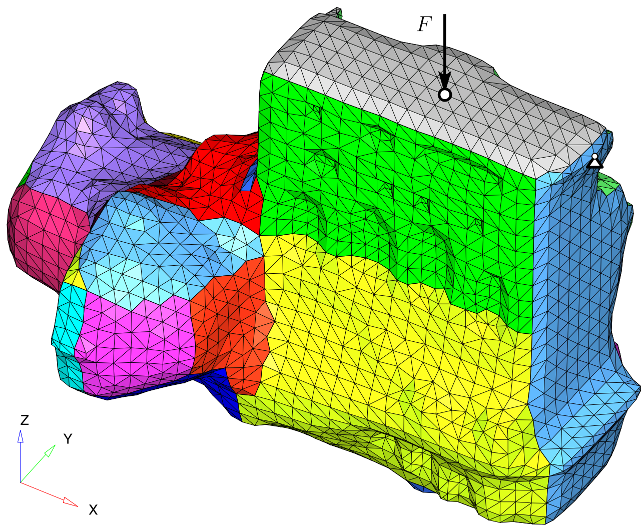


Figure 4: The simplified structural FE model: different colors indicate clustered elements based on the geometric features of the model. These panels are related to a corresponding WB boundary, later on.

In Figure 4, the colors refer to groups of structural elements which are related to a corresponding physical wave based boundary. The structural model was considered to be made of aluminium having the mass density $\rho = 2700$ kg/m^3, the elasticity modulus $E = 7 \cdot 10^9$ Pa, the Poisson's ratio $\nu = 0.3$, the material loss factor $\eta = 0.008$ and an element thickness of $t = 0.001$ m. The model was clamped at three discrete positions and excited by a harmonic point force $F = 1$ N applied at the valve cover, as indicated in the Figure 4. The dynamic behaviour of the structural problem was analyzed in the frequency range up to 1000 Hz by means of a standard FE based system according to Reissner-Mindlin thick plate theory, see Figure 5.

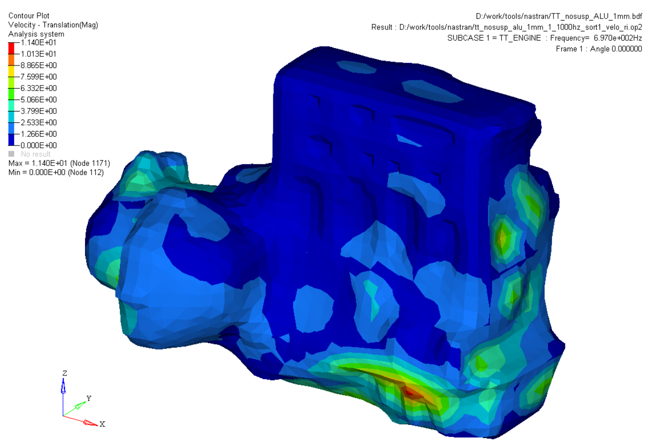


Figure 5: The structural velocity magnitude calculated by the finite element method and plotted at 697 Hz on a deflected shape. Note that the velocities are in [mm/s].

The structural problem was considered to be surrounded by air characterized by its speed of sound $c = 343,8$ m/s and the mass density $\varrho = 1,2$ kg/m^3. The wave based modelling strategy consists in the subdivision of an entire domain Ω^a into a number of convex subdomains. First, the structural FE model was being enclosed by the wave based subdomains until the rectangular domain, see, respectively, Figure 2(b) and 6, was filled. In Figure 2(b), the broken lines indicate interfaces Γ_i between the WB subdomains whereas the colors in Figure 6 refer to an actual wave model partitioning. The wave model was further extended by the six curved subdomains which form the truncation boundary Γ_T, as illustrated in Figure 2(b). Finally, outside the truncation boundary the wave based formulation for unbounded problems was applied [8].

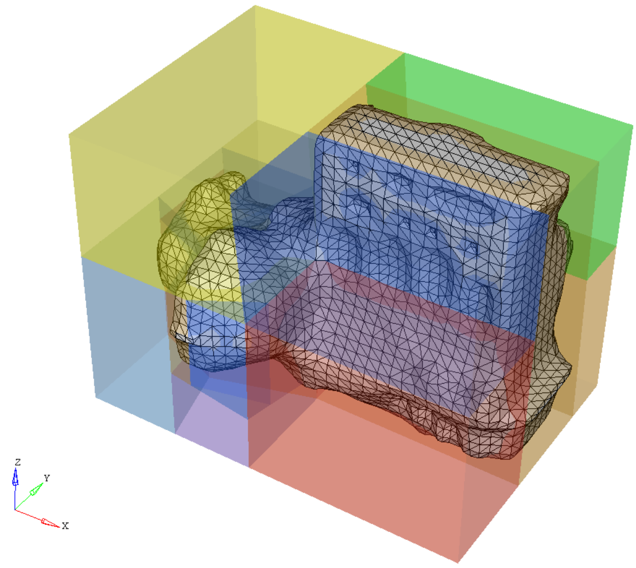
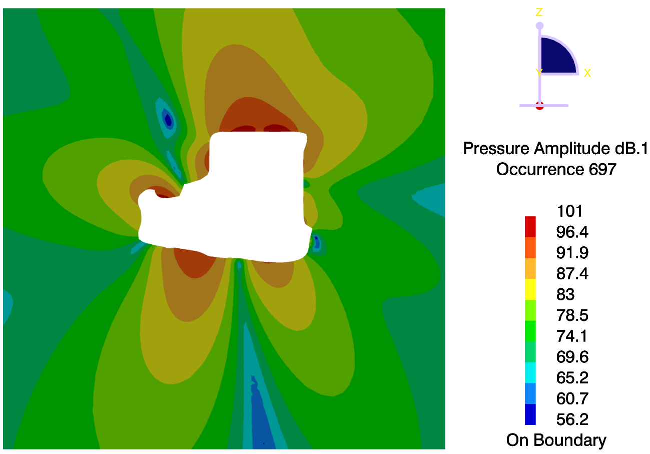


Figure 6: The WB modelling strategy: detailed view of a wave model subdivision. Different colors refer to wave model partitioning. Also note that the six curved elements constituting the Γ_T are omitted.

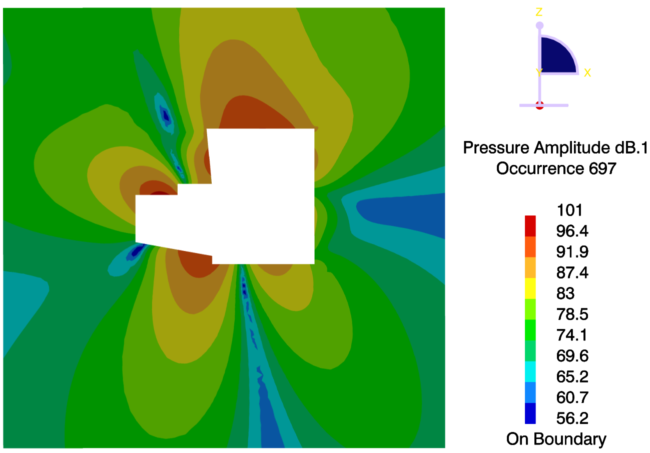
In order to validate the proposed wave based approach a commercial boundary element based software package was utilized. The BE indirect model was based on identical mesh as used for the structural FE analysis, see Figure 4. The primary BE results were postprocessed on a rectangular field point mesh with the dimensions of 2×2 m which intersects the acoustic domain Ω^a parallel to a xz-plane. Figure 7 captures the sound radiation pattern (pressure amplitude in dB re $2 \cdot 10^{-5}$ Pa) predicted at 697 Hz by the boundary element and wave based approach, respectively, which corresponds to a deflection shape as shown in Figure 5. The global sound radiation behaviour shows a good agreement between the both methodologies. Since the field point meshes somewhat vary in the vicinity of the vibrating structure due to the model specific discretization, the near field results are, however, resolved with a slightly different accuracy.

Conclusions

This paper reports on the application of the wave based approach for the analysis of one-way coupled structural-acoustic unbounded problems. The industry-sized validation example demonstrates the capability to tackle industrial real-life problems and illustrates an



(a)



(b)

Figure 7: The pressure amplitude in dB (re $2 \cdot 10^{-5}$ Pa) at 697 Hz predicted by (a) the boundary element method and (b) the wave based technique. Note that the field point meshes slightly differ in the near field region.

enhanced practical applicability via coupling with the standard FE codes. A comparison between the boundary element method and the wave based technique based on the analysis of an engine block-gearbox assembly illustrates the prediction accuracy of the wave based methodology.

Future research focuses on the coupling of the wave based methodology with, respectively, an experimentally acquired data and/or a fully-resolved powertrain simulation results based on a hybrid multi body/finite element time-domain analysis, which incorporates the actual engine dynamics. An ongoing research also addresses the automatization of the MLV approach.

Acknowledgments

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