Properties of bubbles at pressure antinodes in a standing sound field

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Introduction

Cavitation is a complex phenomenon connected with oscillations of gas filled bubbles in liquids. The behavior of cavitation bubbles is well studied in the case of the dynamics of single bubbles as a basis of ultrasonically induced bubble luminescence (sonoluminescence), laser induced bubble luminescence, ultrasonic cleaning and cavitation erosion. The study of collective dynamics is important for the understanding of structure formation in cavitation bubble clouds (acoustic Lichtenberg figures) and of the interaction of bubbles with each other or with a sound field (Bjerknes forces), in particular as a basis for cavitation cleaning and erosion and for sonochemistry.

In the last decade attention has been paid to high amplitude oscillations to reach high temperatures inside a bubble. For spherical bubbles any temperature can be reached. However, due to the required stability of the bubble oscillation, in particular upon collapse, and of its position in the acoustic field, only a limited region of the parameter space spanned by the rest radius of the bubble, R_{n}, acoustic driving pressure, p_{a}, acoustic frequency, f, gas concentration in the liquid, c_∞, static ambient pressure, p_{stat}, temperature of the liquid, T_{l}, chemical composition of the bubble medium, etc., is available for reaching high temperatures in reality.

Limiting factors are mainly given by deviation from the spherical shape, effects of diffusion and instability of the position of the bubble in the pressure antinode. For higher oscillation amplitudes a perturbation of the surface shape of the bubble may occur and may lead to an asymmetric bubble collapse and shape instability [1, 2]. A shape asymmetry strongly suppresses light emission, as has been demonstrated experimentally for laser generated bubbles collapsing near solid boundaries [3], indicating lower temperatures inside the bubble in the case of nonspherical bubble collapse. During bubble oscillation the gas content of the bubble changes due to diffusion of molecules though the bubble surface. This can, under certain conditions, lead to diffusive instability resulting either in the dissolution or the growth of the bubble [4, 5, 6]. Additionally, in standing sound fields, as used for bubble trapping, the primary Bjerknes force drives strongly oscillating bubbles away from the pressure antinode preventing stable trapping [7, 8] and thus reaching high temperatures.

Scope of the investigation

Experiments with transient laser bubbles have shown that the luminescence from bubbles may be increased, when the ambient static pressure is increased [9]. To explore this finding for driven bubbles in a sound field, the idea appeared to investigate bubble oscillations in sound fields under higher than normal static pressure and to calculate the maximum attainable temperatures inside bubbles to establish a possible trend and to learn more about the magnitude of the static pressure effect including stability questions for more realism. Thus the aim of this work is to predict the dependence of maximum temperature in a bubble on ambient pressure considering shape, diffusive and positional stability features in a standing acoustic wave field. Due to the increased stability of small bubbles a sound field of frequency 1 MHz is chosen for the investigations [10]. More precisely, the region of shape and positional stability with possibly diffusive stability, the 'bubble habitat', in the parameter plane made up of the bubble radius at rest, R_{n}, and the driving sound pressure amplitude, p_{a}, at a frequency of 1 MHz is determined for different static pressures, p_{stat}, in the range of 100 kPa to 600 kPa. The temperatures inside the bubble at collapse in this region are calculated.

Bubble model

In the model adopted, a gas filled spherical bubble isolated in an infinite, compressible and viscous liquid oscillates under the action of a sinusoidal sound wave. The model is based on the Keller-Miksis equation [11] simplified by omitting the time delay [12]:

$$\left(1-\frac{\dot{R}}{C_0}\right)R\ddot{R}+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3C_0}\right) = \left(1+\frac{\dot{R}}{C_0}\right)\frac{p_{\mathrm{l}}}{\rho}+\frac{R}{\rho C_0}\frac{\mathrm{d}p_{\mathrm{l}}}{\mathrm{d}t}, \quad (1)$$

$$p_{\mathrm{l}} = \left(p_{\mathrm{stat}}-p_{\mathrm{v}}+\frac{2\sigma}{R_{\mathrm{n}}}\right)\left(\frac{R_{\mathrm{n}}}{R}\right)^{3\gamma} -p_{\mathrm{stat}}+p_{\mathrm{v}}-\frac{2\sigma}{R}-\frac{4\mu}{R}\dot{R}-p_{\mathrm{ac}}(t), \quad (2)$$

$$p_{\mathrm{ac}}(t) = p_{\mathrm{a}}\cos(\omega t).$$

The equation is solved (using Runge-Kutta numerical methods) for the constants: liquid density $\rho = 1000\ \mathrm{kg/m^3}$, surface tension $\sigma = 0.072$ N/m, viscosity $\mu = 0.001$ Pa·s, vapor pressure (water) $p_{\mathrm{v}} = 2.33$ kPa, static pressure $p_{\mathrm{stat}} = 100$ kPa to 600 kPa, polytropic exponent $\gamma = 1.67$, sound velocity in the liquid $C_0 = 1483$ m/s. The dependent variables in the equation are the radius of the bubble, R, and time, t. An overdot means the derivative with respect to time t, thus $\dot{R}$ is the bubble wall velocity and $\ddot{R}$ is the bubble wall acceleration. R_{n} is the bubble radius at rest. The driving sound field $p_{\mathrm{ac}}(t)$ is assumed to be sinusoidal with pressure amplitude p_a and circular frequency $\omega = 2\pi f$, f being the driving frequency, here chosen to 1 MHz in the calculations.

Stability

Only the temperatures in stable bubbles are to be investigated. To this end, the equations used for the shape, diffusion and positional stability will be given in the subsequent sections. When shape and positional stability requirements are fulfilled and dissolving bubbles at saturation gas pressure are excluded, the corresponding part of the parameter space is said to be the bubble habitat.

Shape stability

To test the shape stability a perturbation in the form of spherical harmonics is introduced and it is calculated, whether the time dependent amplitude coefficients, a_l, of the harmonics decay (parametric stability) or grow (parametric instability). The lowest aspherical mode, numbered $l = 2$, is the easiest to grow. For this mode with the coefficient a_2 the differential equation reads

$$\ddot{a}_2 + B(t)\,\dot{a}_2 - A(t)\,a_2 = 0 \tag{3}$$

with the further coefficients A and B as given by [13]

$$A(t) = \frac{\ddot{R}}{R} - \frac{4}{\rho R^3}\left(3\sigma + 2\mu\dot{R}\right), \tag{4}$$

$$B(t) = \frac{3\dot{R}}{R} + \frac{40\mu}{\rho R^2}. \tag{5}$$

In the diagrams below, the instability border is calculated in the plane $(R_\mathrm{n}, p_\mathrm{a})$ for different static pressures, p_stat.

Mass flow stability

A bubble exchanges gas molecules across its interface with the surrounding liquid. Because of surface tension the gas pressure inside a bubble is higher than the partial gas pressure in the liquid and thus a bubble dissolves. If, however, a bubble oscillates nonlinearly in a sound field, the mass flow may be reverted, an effect called rectified diffusion [4, 5]. The mass flow leads to an alteration of the equilibrium radius, R_n, of the bubble and the following equation can be derived:

$$\frac{\mathrm{d}R_\mathrm{n}}{\mathrm{d}t} = F(R, R_\mathrm{n}, ...)\left(\frac{c_\infty}{c_\mathrm{g}} - \frac{\left\langle (R/R_\mathrm{n})^{4-3\gamma}\right\rangle_{T_\mathrm{osc}}}{\langle (R/R_\mathrm{n})^4\rangle_{T_\mathrm{osc}}}\right), \tag{6}$$

$$c_\mathrm{g} = \left(1 + \frac{2\sigma}{R_\mathrm{n} p_\mathrm{stat}}\right) c_\mathrm{sat}. \tag{7}$$

F is a factor not entering the stability criterion and therefore not fully given here. The equilibrium of (rectified) diffusion (no diffusion on average) is given by $\mathrm{d}R_\mathrm{n}/\mathrm{d}t = 0$ or with equation (6) and (7):

$$\frac{c_\infty}{c_\mathrm{sat}} - \left(1 + \frac{2\sigma}{R_\mathrm{n} p_\mathrm{stat}}\right)\frac{\left\langle (R/R_\mathrm{n})^{4-3\gamma}\right\rangle_{T_\mathrm{osc}}}{\langle (R/R_\mathrm{n})^4\rangle_{T_\mathrm{osc}}} = 0. \tag{8}$$

Here, the new variables are the gas concentration of the gas inside the bubble, c_g, the saturation gas concentration in the liquid, c_sat, and the actual given gas concentration in the liquid far away from the bubble, c_∞. An artificial inert gas with $c_\mathrm{sat} = 0.6$ mol/m^3 is taken. This value is derived from nitrogen, but no chemical reactions are allowed. The notation $\langle . \rangle$ means averaging over time and the averaging time is one period of the oscillation of the bubble, T_osc.

Positional stability

In a sound field with its pressure gradients a finite object as a bubble experiences net forces across the bubble surface. In a standing sound field the force is directed along the gradient of the sound field and reads

$$\vec{F}_\mathrm{B} = -\langle \nabla p_\mathrm{ac}\, V(t)\rangle_\tau \quad \text{with} \quad p_\mathrm{ac} = p_\mathrm{a}(\vec{x})\,\sin(\omega t). \tag{9}$$

$\vec{F}_\mathrm{B}$ is the net force averaged over a time span τ of the bubble oscillation, the bubble entering here via its volume $V(t) = 4\pi/3\,R^3(t)$. The bubble is located at $\vec{x}$ where it encounters the pressure amplitude $p_\mathrm{a}(\vec{x})$ with the actual sound pressure varying sinusoidally in time. It is clear that the force depends on the bubble oscillation. Less clear is that the force may change sign when the oscillation gets stronger and stronger [7, 8]. This is due to the nonlinearity of the oscillation that shifts the maximum of the oscillation more and more from the tension phase of the driving sound field into the pressure phase. Thus a force that at low driving keeps the bubble in the pressure antinode may expel the bubble at higher driving. The threshold for this effect is calculated and only the parameters are kept, where the force is directed towards the pressure antinode to maintain the driving pressure amplitude for the bubble.

Bubble habitat

The region in parameter space, where shape and positional stability criteria are fulfilled and diffusive stability may be possible, is called the bubble habitat. In Figure 1 this region is plotted in green color in the parameter plane $(R_\mathrm{n}, p_\mathrm{a})$ for three static pressures of $p_\mathrm{stat} = 100$ kPa, 300 kPa and 600 kPa, saturation gas pressure in the liquid and the adopted sound frequency of $f = 1$ MHz. The linear resonance frequency for $p_\mathrm{stat} = 100$ kPa is $R_\mathrm{n}^\mathrm{lin} = 4.01\ \mu$m, and stable oscillations are only possible below this value (see [14] for similar diagrams at other sound frequencies). It is seen that with higher static pressure the area of stability grows, both to larger bubble radii and larger acoustic pressures.

Maximum temperatures

To find out which bubble in the green areas oscillates the strongest, the bubble oscillations are calculated for a fine mesh of $(R_\mathrm{n}, p_\mathrm{a})$-values starting the individual calculations with the initial conditions $R(t{=}0) = R_\mathrm{n}$ and $\dot{R} = 0$, leaving out the transients and look for the strongest collapse during several periods of the sound field. Actually the temperature is calculated for an adiabatic collapse with the formula

$$T_\mathrm{max} = T_0\left(\frac{R_\mathrm{n}}{R_\mathrm{min}}\right)^{3(\gamma-1)}. \tag{10}$$

T_0 is the ambient temperature, set to 293 K, and R_min is the minimum radius taken from the numerical bub-

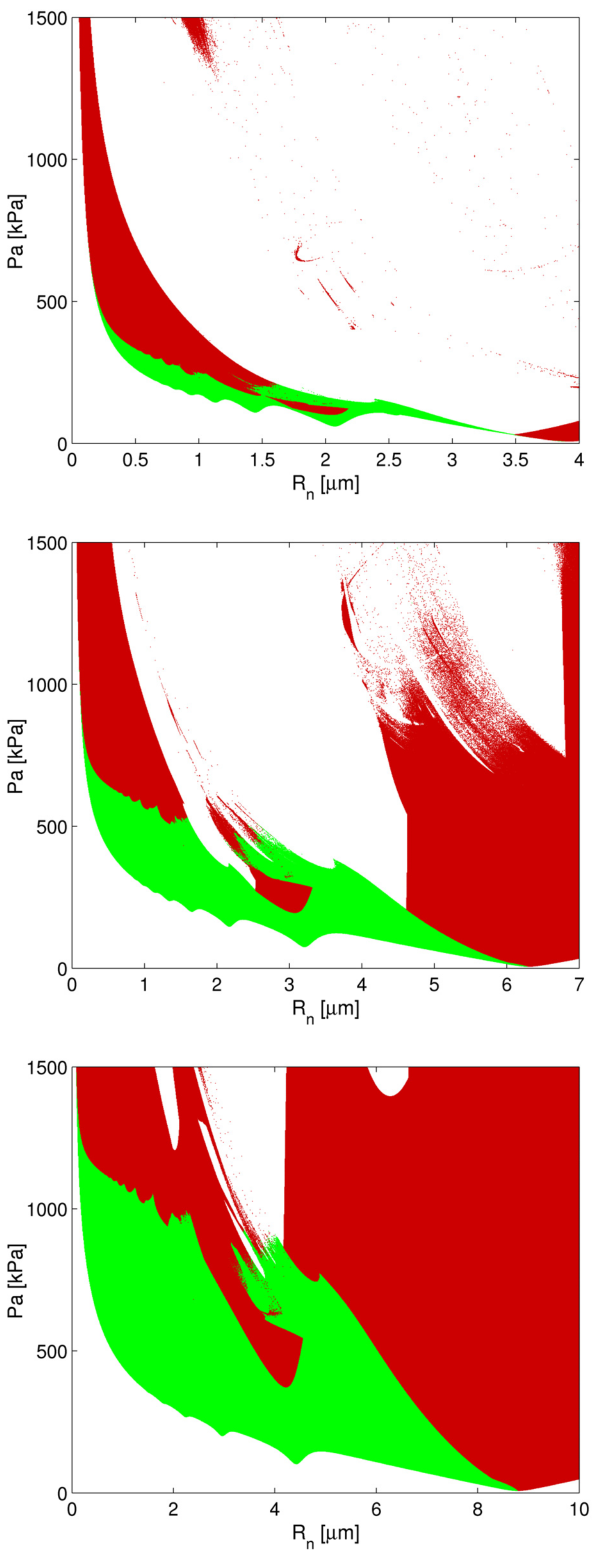

Figure 1: Region of shape and positional stable bubble oscillations in a standing sound field of 1 MHz for three static pressures (green area). In the red area the oscillations are stable with respect to spherical shape, but unstable with respect to position in the pressure antinode. In the area below the lower bound, given by $c_\infty = c_{\rm sat}$, bubbles dissolve.

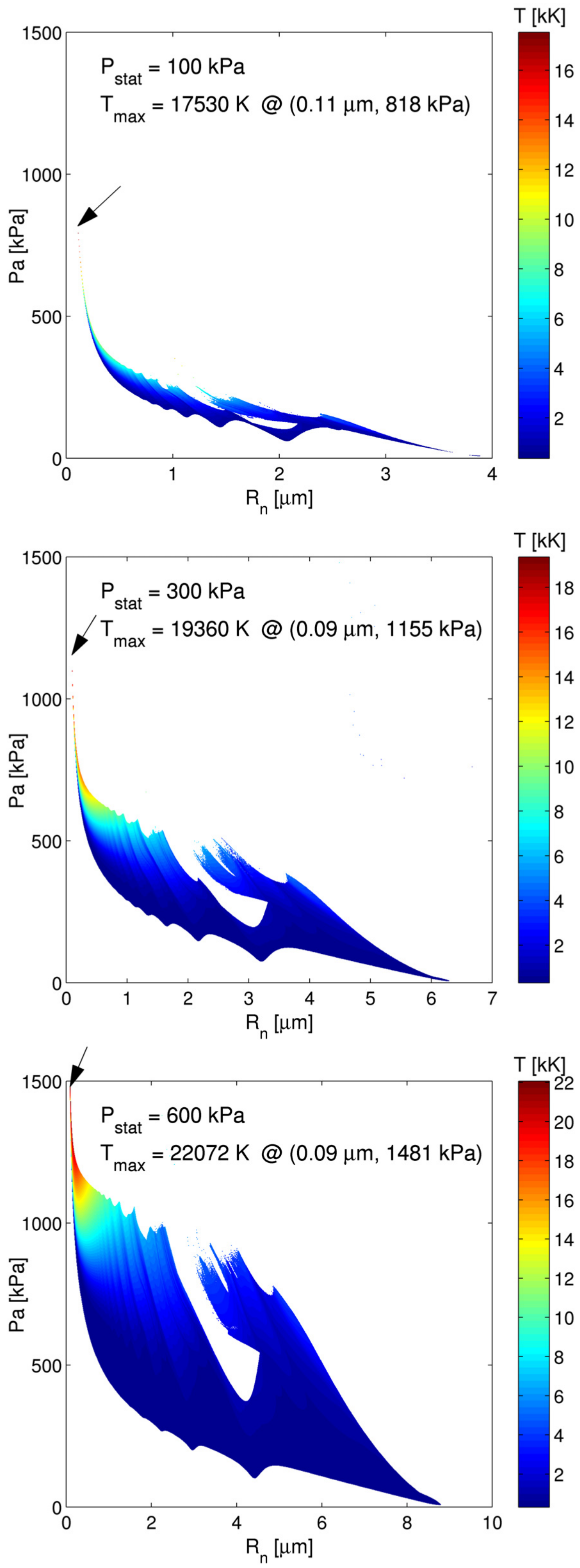

Figure 2: Maximum temperatures calculated for the respective stability regions (colored region) in the parameter plane $(R_{\rm n}, p_{\rm a})$ for a driving frequency of 1 MHz and for different values of the ambient static pressure, $p_{\rm stat}$, as indicated in the diagrams. The absolute maximum temperature for the respective ambient pressure is also noted.

ble oscillation calculation. The maximum temperature obtained for this bubble is plotted in a color code for all $(R_{\rm n}, p_{\rm a})$-values of the habitat area. Figure 2 gives three examples of temperatures obtained, for $p_{\rm stat}$ = 100 kPa, 300 kPa and 600 kPa. The highest temperatures are reached for the smallest bubbles in the mesh of the habitat region for all static pressures investigated up to 600 kPa and are found at the positional stability border.

Conclusions

The influence of the static ambient pressure on the oscillations of bubbles driven at 1 MHz has been investigated numerically. Only oscillations with stability of the bubble with respect to shape and position in the pressure antinode of the standing sound field have been considered. The maximum temperatures reached upon collapse of the bubbles in the ensuing bubble habitat region are calculated with simple adiabatic compression to establish the trend with static pressure. The main results are twofold.
(i) The bubble habitat in the $(R_\mathrm{n}, p_\mathrm{a})$-plane strongly grows with static pressure. It extends to larger bubble radii, R_n, up to the increased linear resonance radius corresponding to the increased static pressure, grossly from about 4 μm at $p_\mathrm{stat} = 100$ kPa to about 9 μm at $p_\mathrm{stat} = 600$ kPa (see Figure 1). It also extends to larger driving amplitudes, p_a, grossly by a factor of two in the interesting giant response region (see [15]) of small bubbles, when going from $p_\mathrm{stat} = 100$ kPa to $p_\mathrm{stat} = 600$ kPa.
(ii) The maximum temperatures calculated at each point of the bubble habitat show the tendency to increase with driving pressure and thus accumulate at the upper (positional) stability (green - red) boundary. The absolute maximum temperature T_am for a given static pressure is encountered for bubbles in the giant response region well below the linear resonance radius. At the expense of higher driving pressures higher absolute maximum temperatures are reached with growing static pressure. The optimal gas concentration in the liquid to approach these values has still to be determined.

The absolute maximum temperatures are obtained at the very small bubble limit of the bubble habitat with a very steep increase in driving pressure. The ultimate resolution has not yet been achieved, but additional calculations in progress indicate that indeed only a very limited range of small bubble sizes is apt of reaching high temperatures. The experimental realization thus may present a challenge.
The trend seems to continue for higher static pressures and will lead to ever increasing temperatures. The corresponding bubbles will get smaller and smaller and the gas concentration in the liquid has to be chosen appropriately to stably install the corresponding bubble oscillations.

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