Vibration Behaviour and Structure-Borne Sound Transmission of a Resiliently Supported Landing

Emre Taskan\textsuperscript{1}, Jochen Scheck\textsuperscript{1,2}, Heinz-Martin Fischer\textsuperscript{1}

\textsuperscript{1} University of Applied Sciences, 70174 Stuttgart, Germany, Email: emre.taskan@hft-stuttgart.de
\textsuperscript{2} STEP GmbH, 71364 Winnenden, Germany, Email: info@steponline.de

Introduction

The expected normalized impact sound level for heavy stairs and landings is presently predicted for less and obsolete constructions and insulation in the latest version of the DIN 4109. Thus the calculation of the standard impact sound level for resiliently supported heavy stairs and landings is according to DIN 4109 – Beiblatt 1 [1] impracticable. By analyzing the structure-borne transmission paths, appropriate acoustical parameters can be related to the involved elements (i.e. flight of stairs, stair landing or decoupling elements) and included into a prediction model, for example EN 12354 [2]. This is the aim of an ongoing research project in cooperation with the STEP GmbH. The vibration behaviour of the landing and the transmission through the resilient layer are of particular importance regarding the whole transmission process as illustrated in Figure 1.

Investigated transmission system

Experimental investigations have been carried out in the staircase test facility. The set-up which is described in more detail in [3] is shown in Figure 2. The landing with dimensions of 2.8 m x 1.3 m x 0.18 (length x width x height) is resiliently supported in the separating wall (24 cm CaSi with density 1800 kg/m\textsuperscript{3}) and a similar second wall (not involved in the transmission) using PUR-Elastomer elements. Like in the building there are two supports in the separating wall. Unlike in buildings there is only one support on the other end of the landing. This turnable set-up was chosen to ensure that the same pressure acts on both resilient layers in the separating wall. Initial measurements of the normalized impact sound pressure level showed a strong dependence on the position of the ISO tapping machine in the frequency range below 1000 Hz (Figure 3).

\begin{equation}
P = \frac{1}{2} \left| \hat{F} \right|^2 \text{Re}\{Y_R\} \quad \text{[W]} \tag{1}
\end{equation}
The force spectrum of the tapping machine from [4] in third octave bands with centre frequency \( f_m \) is given by (2).

\[
| \dot{F} \dot{v} | = 1.85 f_m \quad \text{[N]} \quad (2)
\]

For the hammer mobility \( Y_S \gg Y_R \) as given for the heavy landings considered here the force spectrum is invariant. In contrast the landing mobility is dependant on the excitation position and obviously the reason for the variation of the impact sound pressure level for different excitation positions. In the following the influence of the mobility or more generally said the vibration behaviour is investigated.

**Experimental analysis of the vibration behaviour**

The vibration behaviour of the landing was investigated by means of an experimental modal analysis. The roving hammer method was used with three fixed receiver positions P1, P2, P3 as shown in Figure 2. Two accelerometers were on the landing, one was on the wall to investigate the transmission from the landing into the separating wall. The landing was excited on totally 374 grid positions (10 cm spacing) and the transfer mobilities to the receiver positions measured. Due to reciprocity the receiver positions can be regarded as excitation positions in the visualisation of the vibrations. In the following position P1 on the landing is used as reference position for visualisation of the vibrations and the prediction of the input power later in this paper. The averaged transfer mobilities for receiver positions P1, P2 on the landing and P3 on the stair wall are shown in Figure 4.

![Figure 4: Averaged transfer mobilities for excitation on 374 grid positions to the reference positions P1, P2 on the landing and P3 on the stair wall](image)

The peaks in the average transfer mobilities involving P1 and P2 represent the structural modes which are exemplary shown in Figure 5. The average transfer mobility involving P2 is higher indicating maximum receptiveness at the edges. The first two peaks below 100 Hz represent rigid body vibrations around the longitudinal and transverse axis of the landing. The first natural bending mode occurs at 133 Hz. In the above frequency range the vibration is essentially determined by plate modes of higher order. In addition asymmetrical modes like at 474 Hz occur as a result of the three supports. The average transfer function to the wall is far below the ones for the landing proving the efficiency of the PUR-Elastomer for the sound insulation. However the transfer mobility to position P3 shows significant peaks at the Eigenfrequencies of the landing modes. From this it is clear that the natural landing bending modes govern the transmission. Actually maximum transmission results where landing and wall modes coincide. Thus the modal behaviour is an important effect to consider concerning the interpretation of measurement results and modelling.

![Figure 5: Vibration shapes of the landing from experimental modal analysis](image)

The landing modes are well separated and weakly damped. This is expressed by the modal overlap factor in Figure 6. The measured MOL is significantly smaller than 1 in the frequency range of interest. From this an SEA approach regarding the prediction appears critical.

![Figure 6: Modal overlap factor on the landing compared to the SEA requirement (MOL ≥ 1) according to Craik [5]](image)
Methods for prediction of the landing mobility

Due to the modal behaviour of the landing and interaction with the wall an accurate prediction of the landing mobility is desired for setting up a modal model for the transmission.

In this concern three approaches were followed. An analytical method given by Gardonio/Brennan [6], a numerical approach using a Finite Element Model and an approach involving the mobility of an infinite plate given by [4]. The structure was modelled as a plate with free edges with the input parameters shown in Table 1. Young’s modulus, density and loss factor were taken from independent measurements.

**Boundary conditions**

| Free |

**Material parameters**

| Young’s modulus | 37.5e9 Pa | Measured |
| Density | 2555 kg/m³ | Measured |
| Poisson ratio | 0 | Literature |
| Loss factor | Regression | Measured |

**Geometrical parameters**

| x- length | 2.4 m |
| y- length | 1.3 m |
| Thickness | 0.18 m |

**Geometrical parameters for FEM prediction**

| Exactly modelled |

---

### Table 1: Input parameters for the landing model

#### Analytical model

The analytical model is based on a modal summation of beam functions in x- and y- direction. All boundary conditions can be modelled. The outcome is point and transfer mobilities for any excitation and receiver position given by (3).

\[
Y_{Fe(x_2,y_2)}^{x_1,y_1} = j\omega \sum_{m,n=1}^{\infty} \frac{\Psi_{mm}(x_2,y_2)\Psi_{nn}(x_1,y_1)}{\rho h l x y [\omega_{mn}(1+jn) - \omega^2]} \]

\[
[(m/s)/N] \quad (3)
\]

With this data the vibration shapes of the landing can be visualized for comparison with the measured vibration shapes. From this it is found that the natural plate bending modes are predicted why certainly the asymmetrical modes are missing. In Figure 7 the measured and calculated point mobility at the reference position P1 is compared. In the prediction the peaks are shifted and partly underestimated. Generally there are fewer peaks in the predicted mobility due to the presence of modes resulting from the asymmetrical geometry of the landing.

#### Finite Element Model

The apparent advantage of Finite Element Modelling is that any geometry can be modelled. The landing geometry is asymmetrical and there are block outs for supporting the stairs (Figure 2). With the geometry exactly modelled the agreement with the measured vibration behaviour is considerably better as in the analytical model which can be seen in Figure 8 and Figure 9.

**Figure 7:** Point mobility at P1 measured and calculated using an analytical model [6]

**Figure 8:** Point mobility at P1 measured and calculated using a Finite Element Model

**Figure 9:** Vibration shapes of the landing from Finite Element Model
The asymmetrical modes that were missing in the analytical model are included. The deviation of the measured and calculated Eigenfrequencies is significantly smaller and also the height of the resonance peaks is in good agreement.

**Infinite Plate mobility**

In the simplest approach for modelling the mobility the landing is regarded as an infinite plate (4).

$$Y_m = \frac{1}{8\sqrt{B'm''}} \quad [(m/s)/N]$$  \hspace{1cm} (4)

Input parameters are the bending stiffness and the mass per unit area. The mobility of an infinite plate is real and independent from the position. Figure 10 shows the comparison with the measured point mobility. The discrepancies at the resonance and anti-resonance frequencies are up to 15 dB as a result of the low damping. However the infinite plate mobility gives a good frequency average of the true mobility.

**Injected Power**

The power injected into the landing when excited by a structure-borne sound source is deemed to be a useful quantity regarding the physical description of the whole transmission chain as illustrated in Figure 1. With the measured and calculated mobilities as shown in Figures 7, 8, 10 and the force spectrum of the tapping machine (2) the power injected into the landing was predicted with (1). For simplification the mobility at position P1 was used instead of an average for the five hammer positions. The results are shown in Figure 11 in third octave bands. With the mobility from the analytical model the differences are up to 10 dB in the frequency range up to 1 kHz which is unsatisfactory. Generally the power is underestimated. Using the mobility from the FE model the predicted input power is within ± 5dB in the relevant frequency range from 100 Hz upwards which is acceptable. The prediction using the FE model is certainly the best choice in setting up a modal model for the transmission. With the infinite plate mobility the agreement is ± 5dB for f > 200 Hz. The big discrepancies observed in narrow bands average out in third octave bands.

![Figure 10: Point mobility measured at P1 and calculated for an infinite plate](image)

**Conclusion**

The vibration behaviour of a resiliently supported landing was analysed in the staircase test facility. It is found that the maximum sound transmission into the wall occurs at the few landing (and wall) Eigenfrequencies. Thus the modal behaviour is an important effect to consider concerning the interpretation of measurement results and modelling of the transmission process. The characterisation of decoupling elements like the PUR-Elastomer considered here can not be seen independent from the whole system consisting of landing and stair wall. Therefore a defined reference situation is required. The FE simulation is in good agreement with the measured results. It is intended to advance the FE model in order to simulate the transmission from the landing through the resilient layer into the wall to enable parameter studies which could help in generalising measured results.

**References**