About the Threshold of the Transient Ultrasonic Cavitation in the Cavitational Noise at Different Frequencies

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1. Introduction

The availability and the increasing practical use of different ultrasonic frequencies for ultrasonic cleaning in the range of 20 - 150 kHz raise the questions

- what is the minimum ultrasonic *power* necessary to generate transient cavitation for the different frequencies in this range?
- how does the minimum ultrasonic *power* depend on the frequency?

For the minimum of acoustic *pressure* P_T necessary to generate transient cavitation in a given liquid under given external conditions (static pressure, temperature) the term “threshold pressure for transient cavitation” is generally accepted in the literature /1-3/.

Esche compared the inception of cavitation in water, generated and measured by different methods, between 3-3300 kHz /4/.

In this paper we compare the experimentally determined inception of transient cavitation for different frequencies as function of the electric effective power applied to an ultrasonic transducer area I_E (W/cm^2) with a theoretically derived frequency dependence of the cavitation threshold. The inception of transient cavitation was determined experimentally from the measured curves for the cavitation noise level L_p [dB, rel. 1 W/m^2] of the squared acoustic pressure as function of the electric intensity I_E (Fig.'s 6, 7): In the logarithmical scale the onset of their linear behavior with slope ~1 at increasing electric intensity indicates the inception of transient cavitation. Above this onset further increase of electric effective power input results in the equal increase of the cavitation noise level.

The theoretically derived frequency dependence of the cavitation threshold, on the other hand, is based on

- Apfel´s equation /1/ for the frequency-dependent transient cavitation threshold P_T and on
- the assumption of a constant value for Z in the relation

$$I_E \sim I_A = P_{A,rms}^2/Z \qquad [1]$$

between the acoustic intensity I_A [W/cm^2], time averaged acoustic pressure P_A [Pa] and acoustic impedance Z [Pa sec/m].

2. Threshold pressures for bubble growth

There are thresholds for the acoustic pressure in a liquid of surface tension σ at ambient pressure P_o (100 kPa in this paper) which have to be exceeded to let a bubble grow. In a theoretically pure liquid and at P_A = 0 all bubbles would dissolve because of σ, of course.

(1) Blake threshold pressure amplitude $P_A = P_B$, sufficient for the growth of a bubble with initial radius $R = R_B$ at $P_A = 0$ /5/:

From the condition of mechanical equilibrium

$$(P_o + 2\sigma/R_B)(R_B/R)^3 = P_o - P_A + 2\sigma/R \qquad [2]$$

and searching in Equ. [2] for the minimum of external pressure P_A– P_o (P_A being the negative peak value of acoustic pressure) for bubble growth using $\partial(P_o\text{-}P_A)/\partial R = 0$ one obtains the Blake threshold P_B:

$$P_A = P_B = P_o + (8\sigma/9)\,[3\sigma/2(P_o + 2\sigma/R_B)R_B^3]^{1/2}. \qquad [3]$$

Obviously this approach does not consider bubble dynamics at all. Therefore this threshold is independent of the acoustic frequency. Fig. 1 shows the main result: the smaller the initial radius R_B and the higher the surface tension σ the higher the acoustic pressure has to be for bubble growth.

(2) Taking into account the dynamic effect of rectified diffusion the threshold pressure amplitude $P_A = P_D$, sufficient for the growth of a bubble with initial radius R_D at $P_A = 0$ by “acoustic pumping” of dissolved gas into the oscillating bubble is given by /5, 6/:

$$P_A = P_D = P_o[3(1+X_D)\text{-}X_D][1\text{-}(f/f_D)^2][1\text{-}C+X_D]^{1/2}/[6(1+X_D)]^{1/2}. \qquad [4]$$

with $X_D = 2\sigma/(P_oR_D)$, $C = C_i/C_o$, using C_i as the initial uniform and C_o as the saturation gas concentrations in the liquid, and with

$$f_D = [(3P_o/\rho)(1+2X_D/3)]^{1/2}/(2\pi R_D), \qquad [5]$$

the resonance frequency of the bubble in its initial state ($R = R_D$).

Equ. (4) is obtained for a bubble oscillating under alternating acoustic pressure with peak value P_A. It uses the equations for the inward and outward diffusion of dissolved gas, includes inertial terms but neglects adiabatic and dissipative effects of bubble dynamics.

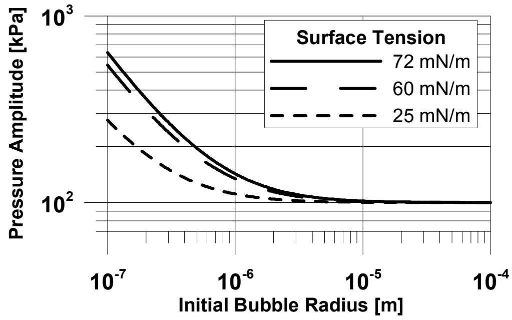

Fig. 1: Blake's threshold pressure amplitudes P_B as a function of the initial bubble radius R_B for liquids of different surface tension.

Fig. 2 reveals that with increasing acoustic frequency up to $f = f_D$ lower but beyond $f = f_D$ higher acoustic pressures are necessary to initiate bubble growth by “acoustic pumping” or rectified diffusion. An additional initial effect (until R_D increases up to the radius of resonance R_r) not taken into account in Equ. (4) is the accelerated growth by the movement of bubbles driven by acoustic forces into regions of local acoustic pressure maxima. For $R_D > R_r$ the same forces push the bubbles out of these regions and slow down growth.

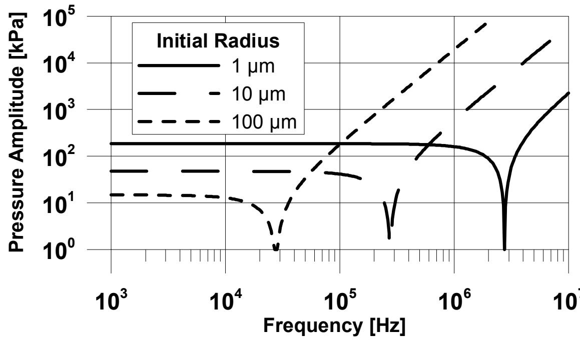

Fig. 2: Threshold pressure amplitudes P_D for growth by rectified diffusion (Equ.[4],C_i/C_o=1,σ=72 mN/m), minima at bubble resonances.

3. Threshold pressure P_T for transient cavitation

Apfel derived an approximated analytic equation for the transient cavitation threshold pressure amplitude $P_A = P_T$ /1/. Considering Rayleigh´s theory for collapsing empty cavity in a liquid, he asked for the maximum radius R_{max}, which could be reached before the collapse of a bubble under alternating pressure, as a function of its amplitude P_A and he found:

$$R_{max} = [2/(3\pi f)]\,(P_A\text{-}P_o)[2/(\rho P_A)]^{1/2}[1+2(P_A\text{-}P_o)/(3P_o)]^{1/3}. \qquad [6]$$

This R_{max} does not depend on initial R_o but on acoustic frequency f! Lauterborn found /7/ that the maximum radius R_{max} up to which a bubble in water with initial radius R_o at P_A= 0 must expand during one acoustic period to reach just the threshold of supersonic collaps velocity is: $R_{max} \approx 2.3R_o$. [7]

Thus, a threshold of acoustic pressure for transient cavitation can be defined as the pressure $P_T = P_A$ at which $R_T = R_{max}$ reaches $2.3R_o$. The derivation of Equ. [6] was based also on the assumption that the start-up or “inertial” time t_I, up to which 75% of the ultimate expansion velocity of the bubble in the negative pressure region are reached, is much shorter than the acoustic period T_A: $t_I < T_A/5$. This finally results in the condition that Equ.(6) holds only for bubbles of initial radii R_o smaller than the radius R_I reached by the bubble within t_I:

$$R_o < R_I \approx [(P_T - P_o)/\rho]^{1/2}/(4f). \qquad [8]$$

Fig. 3 shows the dependence of the acoustic frequency f at threshold on the threshold pressure amplitude P_T for 3 initial bubble radii R_o. It reveals that above of P_T ~ 200 kPa the threshold is already determined by the additional condition [8] – i.e., R_I has to be used in Equ. [7] instead of R_o.

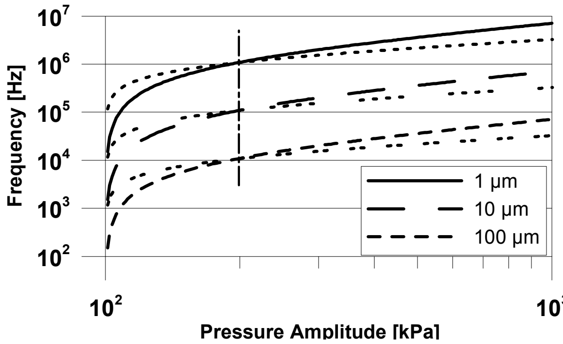

Fig. 3: Acoustic frequency f at threshold as a function of the acoustic pressure P_T for R_o = 1, 10, 100 µm. For P_T > 200 kPa the curves using R_I (**....**) instead of R_o (___) define the threshold (s. text).

Fig. 3 shows that for a given bubble the threshold pressure for transient cavitation increases with frequency and at a given frequency this threshold pressure becomes higher for bigger initial bubbles. For P_T > 200 kPa R_I determines the pressure threshold. Apfel /1/ and Young /2/ give the threshold of P_T for P_T above 1100 kPa as determined by R_I instead of R_o. Their result is obtained, if a factor of 1 instead of 2.3 is applied in Equ. [7] in case of R_I.

Using Equ. [1], Equ. [7], $P_T = P_A$ and inserting $P_{A,rms} = (I_T Z)^{1/2}$ for a plain wave in Equ. [6] we can solve Equ. [6] for the frequency $f = F(I_T)$ at which the threshold of acoustic intensity I_T for transient cavitation is just reached for given initial bubbles with initial radii R_o as parameters. Inverting this relation one gets finally the aspired dependence threshold of the acoustic intensity I_T on the acoustic frequency:

$$I_T = F^{-1}(f) \qquad [10]$$

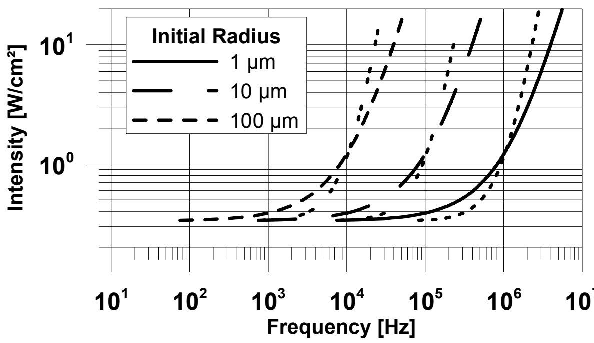

Fig. 4: Acoustic intensity I_T necessary for cavitation threshold as a function of frequency f for R_o= 1, 10, 100 µm. For P_T > 200 kPa the curves with R_I (....) instead of R_o (___) define the threshold (s. text).

Fig. 4 shows that for a given bubble the threshold intensity for transient cavitation increases with frequency and that at a given frequency this threshold intensity is slightly higher for bigger initial bubbles.

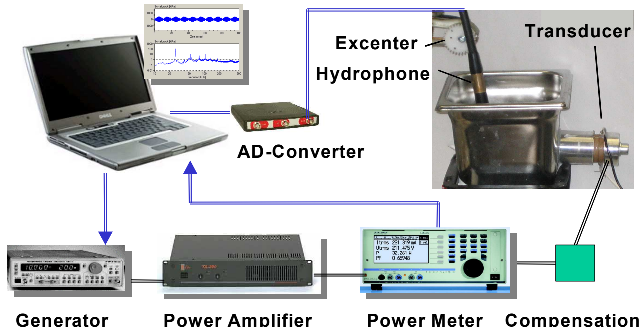

Fig. 5: Experimental setup for cavitation noise level measurement.

Now these predictions derived from Apfel's model will be compared with experimental results. Fig. 5 shows the experimental setup.
The hydrophone measures the noise level between harmonic frequencies /8/ and was moved by an excenter for spatial (25 mm) and temporal (5 s) averaging. The medium was deionized water with wetting agent at a temperature between 20-30°C. For more details of the measurement of the noise level see /8/.
As an example Fig. 6 demonstrates for f = 45 kHz the characteristic distribution of the measured points for the noise level as function of the electric effective power I_E scaled to the transducer area. A running average was used to get a continuous curve. The dashed lines mark the two different slopes below and above the onset. At $I_E \geq 2.3$ W/cm² there is the onset of a linear behavior with slope ~1 with increasing effective power indicating the inception of transient cavitation - being also visible in and audible from the bath.
Fig. 7 shows the averaged curves obtained for the 3 frequencies 27, 45 and 73 kHz. Considering the standing wave pattern in the bath, doubling the acoustic pressure near the reflecting wall and the efficiency factor of 85% the 3 different onsets for the intensity obtained are 0.51, 0.64 and 1.1 W/cm² for 27, 45 and 73 kHz, respectively.
Fig. 8 shows the comparison between the intensity threshold for transient cavitation I_T as a function of the acoustic frequency f from Equ.[10] and the 3 points obtained experimentally. For this small frequency window the curve through the experimental values matches well the Apfel model if one assumes an initial bubble radius of 12 µm. In principle the comparison has to be carried out for a given ensemble of the bubble size distribution, of course.

Literature:

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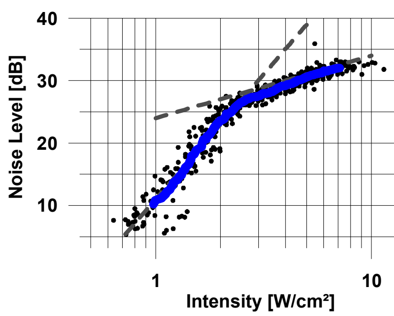

Fig. 6: Measured (points) and averaged (line) noise levels L_p at given intensities I_E for f = 45 kHz. Dashed lines mark the two main slopes.

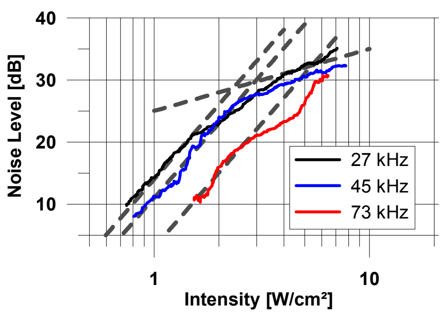

Fig. 7: Averaged noise levels L_p at given intensities I_E for 27, 45 and 73 kHz. The slopes for the 3 cases are marked by dashed lines.

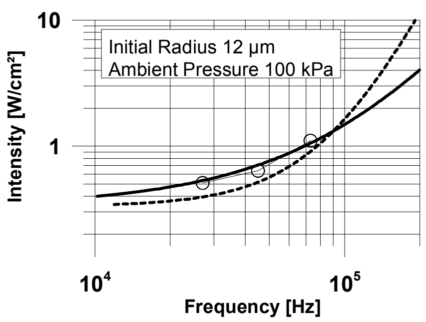

Fig. 8: Comparison of 3 experimental intensity thresholds for transient cavitation with the theoretical threshold from Equ. [10].