# Modal analysis of a recorder: Experiment and simulation

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## Introduction

The recorder is one of the most popular musical instruments. Requiring neither a complicated embouchure nor a difficult handhold, recorders are often used as first instrument in musical education. Nevertheless, the physics of the recorder is not completely known and there is a broad field of research on sound formation, propagation and radiation. Herein, we present a finite element model of the recorder's fluid in infinite space. Prerequisite is a finite element model of the structure of the recorder. Both models are validated by comparing the results of experimental and simulated modal analysis for the structure and by comparing the resonance frequencies from experiments and simulations of the fluid, respectively.

In our contribution, we first describe the development of the model of the structure. Then we present the investigation of the fluid. In both cases, we describe the experiments, the development of the finite element models, and the validation of the models.

## Modal analysis of the structure

To investigate the dynamical properties of the recorder, we performed a modal-analysis experiment. That means, we determined the eigenfrequencies and studied the shapes of the corresponding eigenmodes. The data, which we obtained in this way, were used as basis for the validation of the finite element model of the structure.

The experimental modal analysis was performed by means of the STAR System, a special device which determines the dynamic property values from the response data. For this aim, the recorder was hung up horizontally. The *single input single output* version was chosen, means response was measured at a fixed position while the structure was excited at different points. The structure was activated to swing by an impact hammer and the response signal was measured with an accelerometer. In a first approach, we used a tri-axial accelerometer. However, due to its weight, it puts the instrument out of tune too much. Thus, we decided to measure the x and y directions separately.

Our measuring data agreed to a large extent with the expectations, a survey of the eigenfrequencies is given in table 1. We detected pairwise bending modes, that means natural frequencies for x and y direction are close to each other. Moreover, we observed torsion modes as well as longitudinal modes. The latter were not exactly reproducible. For that reason, they are not included in table 1. Finally, we found radial modes at higher frequencies.

Comparing with the experimental data, we developed the finite element model of the recorder's structure. Here, we had to introduce several simplifications: In the simulation, we assumed that the recorder consists of only one part, that means we neglected the three-part structure (head joint, middle joint, and foot joint) and the plug connections. A rough estimate showed that these connections have only a weak influence on the natural frequencies. Furthermore a cavity in the head joint is not included in the head joint model. The additional mass affects only slightly the natural frequencies. The difference does not exceed 2%, so that the influence of the cavity can be neglected in the comparison with the measurements. Figure 1 shows the complete finite element model. It has about 110000 degrees of freedoms.

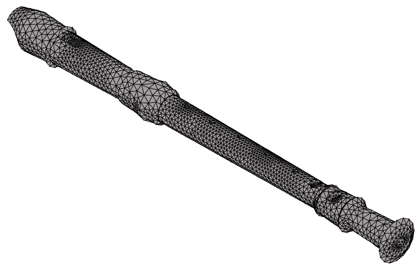

**Figure 1:** The alto recorder as finite element model.

Yet, the Young´s modulus of the synthetic material which the recorder is made of was not known. It influences the natural frequencies but not the shape of the oscillation modes. Avoiding the destruction of the recorder, we determined the Young´s modulus by comparing the natural frequencies of experiment and simulation. For the adjustment, only the middle joint was considered, since the smallest simplifications had to be made in the construction of its model. The obtained natural frequencies are given in table 1. Regarding the simplifications inherent in the simulation and the width of the frequency interval, which ranges from 350 and 1600H$z$, we consider the quality of the finite element model as fully satisfactory.

## Resonance frequencies of the air column

The finite element model, which simulates the behaviour of the fluid inside the recorder, that means the air column, and the fluid around the recorder, was constructed starting from the model of the structure. Experimental data and finite element calculations were again compared

| mode | $f_m$ [Hz] | $f_s$ [Hz] | $d_a$ [Hz] | $d_r$ [%] |
|---|---|---|---|---|
| 1 | 164.87 | 164.61 | 0.04 | 0.02 |
| 2 | 170.78 | 171.51 | 0.73 | 0.43 |
| 3 | 459.18 | 490.95 | 31.77 | 6.47 |
| 4 | 472.22 | 506.85 | 34.63 | 6.83 |
| 5 | 762.72 | 788.04 | 25.32 | 3.21 |
| 6 | 907.96 | 966.17 | 72.66 | 8.00 |
| 7 | 929.33 | 994.37 | 80.27 | 8.64 |
| 8 | - | 1393.5 | - | - |
| 9 | 1380 | 1530.0 | 176.9 | 12.82 |
| 10 | 1440 | 1570.4 | 159.5 | 11.08 |
| 11 | 1470 | 1886.0 | 435.7 | 29.64 |
| 12 | 2020 | 2213.0 | 233.9 | 11.58 |
| 13 | 2060 | 2268.2 | 251.6 | 12.21 |
| 14 | - | 2740.3 | - | - |

**Table 1:** Comparison experimental and computed natural frequencies ($E = 2800\,\text{MPa}$). $f_m$ - measured frequency, $f_s$ - simulated frequency, $d_a$ - absolute deviation, $d_r$ - relative deviation

to learn about the performance of the model.

The measurement setup was similar to a setup for input impedance measurements for reed woodwind instruments developed by J. Backus [3]. In our case, the data of interest were the resonance frequencies which also could be detected with this setup. In comparison to the real playing of the recorder, several simplifications could not be avoided. In particular, there was no air stream during the measurement procedure.

Figure 2 shows the simulation model with finite and infinite elements. To model the infinite space, a sphere is put around the fluid. Finite elements are used inside this sphere. Its surface represents the infinite space and is covered with infinite elements. Our finite element model includes 67000 degrees of freedom. Also in our simulations, the air stream was not taken into consideration.

The values of the natural frequencies, which were obtained by experiment and simulation, are presented in table 2. Although our simulations do not have any adjustable parameters, and in spite of the simplifications made, measured and computed values do not differ by more than roughly 2% from each other. Thus, the finite element model developed, is well appropriate for the computation of the eigenfrequencies.

The simulations of structure and fluid, which are described above, were performed independently from each other. To study possible interactions of structure and fluid, one would have to find out whether or not eigenmodes of the fluid can activate oscillations of the structure. However, the fluid can only activate radial modes of the structure. Due to their high natural frequencies, they can be activated only by higher harmonics of oscillations of the fluid. Thus, comparing the eigenfrequencies of the structure and the fluid, one can argue that the interaction between structure and fluid can be neglected.

This model can now be utilized for further simulations, in particular for comparing the sound radiation properties of differently constructed recorders.

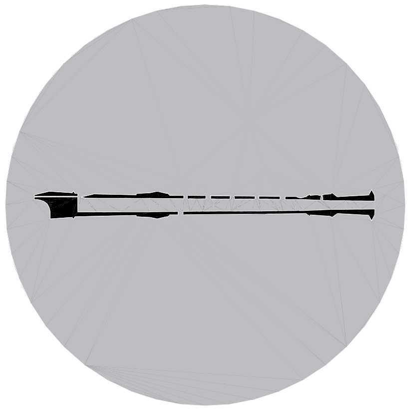

**Figure 2:** The fluid inside and around an alto recorder as simulation model with finite and infinite elements.

| note | n | $f$ [Hz] | $f_m$ [Hz] | $f_s$ [Hz] | $d_r$ [%] |
|---|---|---|---|---|---|
| 65 | F4 | 350.82 | 346 | 351.45 | 1.58 |
| 66 | | 371.68 | 368 | 374.54 | 1.78 |
| 67 | G4 | 393.78 | 389 | 398.21 | 2.37 |
| 68 | | 417.19 | 429 | 425.00 | -0.93 |
| 69 | A4 | 442.00 | 437 | 449.13 | 2.78 |
| 70 | | 468.28 | 464 | 474.33 | 2.23 |
| 71 | B4 | 496.13 | 493 | 497.68 | 0.95 |
| 72 | C5 | 525.63 | 524 | 531.46 | 1.42 |
| 73 | | 556.89 | 560 | 570.57 | 1.89 |
| 74 | D5 | 590.00 | 595 | 602.56 | 1.27 |
| 75 | | 625.08 | 623 | 638.99 | 2.57 |
| 76 | E5 | 662.25 | 669 | 681.85 | 1.92 |
| 77 | F5 | 701.63 | 708 | 709.38 | 0.19 |
| 78 | | 743.35 | 752 | 769.09 | 2.27 |
| 79 | G5 | 787.55 | 803 | 815.25 | 1.53 |
| 80 | | 834.38 | 850 | 859.74 | 1.15 |

**Table 2:** Comparison between experimental and computed data. note - MIDI note number, n - name, $f$ - frequency, $f_m$ - measured frequency, $f_s$ - simulated frequency, $d_r$ - relative deviation

# References


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