Methods to characterize the acoustic properties of periodic surfaces

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Introduction

Periodic surfaces scatter and absorb sound. Periodic gratings create a near and a far field, which differ in strength according to the structure and composition of the surface.
If the sound field in front of the periodic surface can be calculated, the absorption coefficient may be estimated, too. Thus the question arises, whether periodic structures can be characterized by conventional measuring methods applied in case of holohedral absorbers.
In the context of this work it has been investigated which kind of measuring methods allow the characterisation of periodic components with regard to their scattering or absorptive properties. Is it possible to examine the near and far field separately in case of directional, random or normal sound incidence? Is it possible to analyze the scattered waves separately from the geometric reflection? What is the precision and accuracy of these methods?

Theoretical Investigations

Investigations are based on the assumption of a one-dimensional periodic surface with stripes of alternating admittances A1 and A2, as well as a period length Λ_x (cf. Figure 1). A plane wave with wave number k_0 hits this surface under pole angle Θ and azimuth angle Φ.

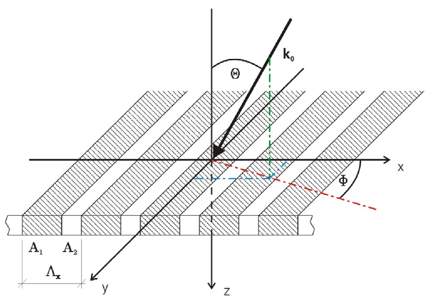

Figure 1 Infinitely expanded, periodic surface area in x-direction, cf. [2].

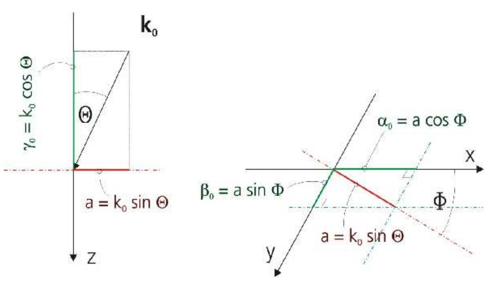

Figure 2: Calculation of wave numbers in the three-dimensional coordinate system, cf. [2].

The sound field in front of the surface is composed of an incident and reflected sound field according to equation (1).

$$p_{in}(x,y,z) + p_{ref}(x,y,z) \tag{1}$$

The reflected sound field is assumed according to the approach of Lord Rayleigh [1] who suggested a composition of an infinite number of harmonic waves (eq. 2). $e^{j\omega t}$ is assumed.

$$e^{j(\alpha_0 x+\beta_0 y-\gamma_0 z)} + \sum_{m=-\infty}^{\infty} R_m \cdot e^{j(\alpha_m x+\beta_m y+\underline{\gamma_m} z)}$$

$\alpha_0 x, \beta_0 y, \gamma_0 z$: Wave numbers of incident sound wave in x-, y- or z-direction (2)

$\alpha_m x, \beta_m y, \gamma_m z$: Wave numbers of reflected sound wave in x-, y- or z-direction

R_m : scattering amplitudes

Concerning the reflected sound field, the waves adopt the form of the periodic surface in x-direction (eq. 3, [3]). The boundary condition and the Helmholtz-Equation are invariant in y-direction. Therefore, the y-component of the wave number of the reflected sound field is equal to the one of the incident sound wave (cf. eq. 4). The propagation of the sound field in z-direction away from the surface can be calculated via the Helmholtz equation and is described by wave number γ_m (eq. 5, [2]).

$$\alpha_m = \alpha_0 + m\frac{2\pi}{\Lambda_x} \tag{3}$$

$$\beta_m = \beta_0 \tag{4}$$

$$\underline{\gamma_m} = \pm\sqrt{\vec{k}_0^{\,2} - \alpha_m^{\,2} - \beta_0^{\,2}} \tag{5}$$

If the real part of γ_m is greater than zero, a propagating wave $e^{j\gamma_m z}$ results. Also if the imaginary part of γ_m is less than zero, evanescent waves $e^{\gamma_m z}$ result. Thus, the propagating constant γ_m indicates whether the reflected sound scatters into the far field or not. In order to determine the energy content of the scattered sound field, the amplitudes R_m must be retrieved. This is accomplished by solving the linear system of equations cf. Eq. (6) which is established by means of the known admittance as boundary condition (eq. (6), [2]).

$$\sum_{m=-\infty}^{\infty} R_m(\omega\rho_0 a_{n-m} + \delta_{m,n}\underline{\gamma_m}) = -\omega\rho_0 a_n + \delta_{0,n}\gamma_0 \qquad (6)$$

with

$$\delta_{m,n} = \begin{cases} 1 \text{ if } n=m \\ 0 \text{ if } n\neq m \end{cases} \quad \text{and} \quad \delta_{0,n} = \begin{cases} 1 \text{ if } n=m \\ 0 \text{ if } n\neq m \end{cases}$$

ω : angular frequency , ρ_0 : density of air, a_n : Fourier coefficients describing the boundary condition at z=0

The absorption coefficient is depends on Θ and Φ. It can be calculated according to equation (7) cf. Takahashi [3].

$$\alpha_{\Theta,\Phi} = 1 - \sum_{\mathrm{Re}\{\underline{\gamma_m}>0\}} |R_m|^2 \cdot \frac{\underline{\gamma_m}}{\gamma_0} \qquad (7)$$

Only the space harmonics which are actually radiated into the far field, are used for calculating the absorption coefficient.

Results of Numeric Modelling

In order to demonstrate the excess sound absorption of a periodically striped structure, its sound absorption spectrum calculated according to eq. (7) will be compared with the area average absorption coefficient (cf. Figure 3). The sound absorption of the striped structure clearly exceeds the area averaged one. At a certain point in frequency the absorption coefficient drops. From this point on scattering (additional to the geometrical reflection) occurs. The cut-on frequency is only dependent on the geometry of the structure, whereas the magnitude of the reflection coefficients R_m is dependent on the boundary condition at z=0. Thus, the smaller the period Λ_x the later in frequency domain scattering occurs.

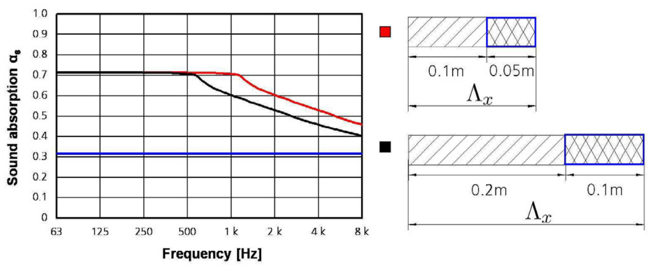

Figure 3: Sound absorption coefficient for random sound incidence calculated via eq. (7) and as area average. The structure consists of two stripes characterised by their specific acoustic admittance independent of frequency (a_1=0 and a_2=0.5).

The effect of the admittance difference between the rigid and absorbing stripes is shown in Figure 4. The higher the admittance difference the higher the effect on sound absorption.

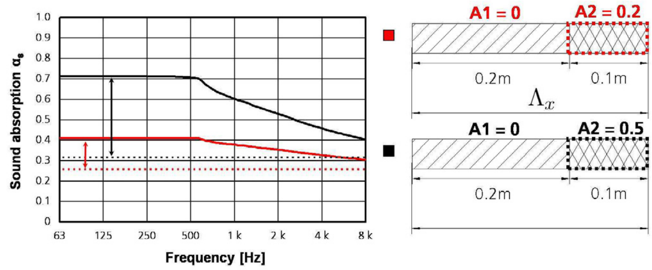

Figure 4: Sound absorption coefficient for random sound incidence calculated via eq. (7) and as area average.

Experimental Investigations

Two measuring methods for random and directional sound incidence are represented in the following. Further experimental investigations on normal, random or directional sound incidence are given in [5].

Random Sound Incidence

It is possible to separate the geometric reflection from the scattered ratio by means of the scattering coefficient method according to [4]. For this purpose, a circular specimen of the periodic structure is pivot-mounted in a reverberation room (cf. Figure 5).

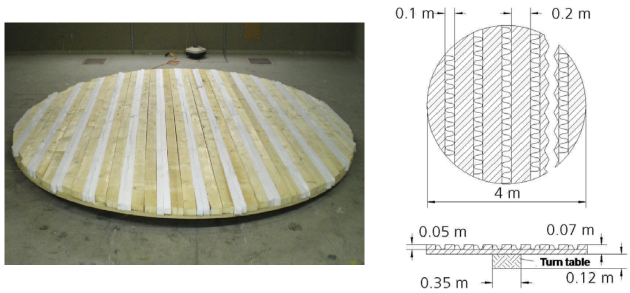

Figure 5: Pivot-mounted structure in the reverberation room (left). Diagram of the test set-up (right)

The impulse response is measured in this position. Then, the specimen is rotated by 45° and once again the impulse response is measured at identical positions of loudspeaker and microphone. Figure 6 shows that the early part of the impulse response is coherently super-imposed from the direct sound, whereas the impulse responses of the late part diverge.

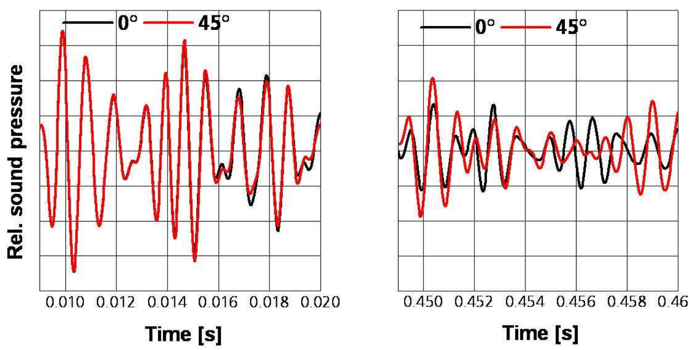

Figure 6: Early parts of impulse responses from the direct sound (left). Late parts of the impulse responses (right)

This effect occurs, since the geometric reflection always develops at the same time independent of the position of the specimen, whereas the scattered part is changed by rotating the specimen. This effect is used to separate the geometric reflection from the scattered waves.

If the impulse response is averaged while the specimen is continuously rotating, the scattered sound in the late part of the impulse response is averaged by destructive interferences.

The resulting curves show that an absolute scattering coefficient can be determined from the measured absorption coefficients of rotating and non-rotating specimens, allowing the quantification of the structure with regard to the scatter properties (see Figure 7).

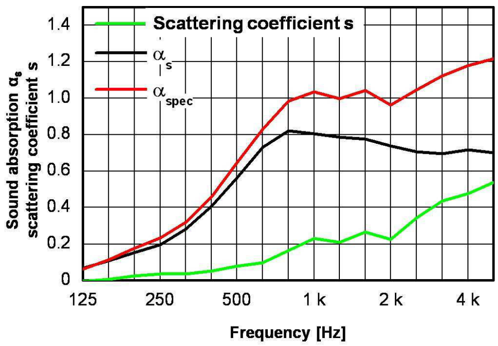

Figure 7: Scattering coefficient determined from absorption coefficients of rotating specimen α_{spec} and non-rotating specimen α_s

Directional Sound Incidence

The following measuring method allows the establishment of scatter diagrams, which explain how a plane wave is reflected at a structure.

For this purpose, a specimen is suspended from the ceiling by means of steel cables in the anechoic room. It is irradiated by a measurement signal in the specimen centre from a loudspeaker with omni-directional radiation characteristic at a distance of 8 m. A semi-circular aluminium rail with a radius of 5 m, where a microphone is fixed in the centre, is installed around the specimen. This microphone is positioned in steps of 2 degrees (see Figure 8).

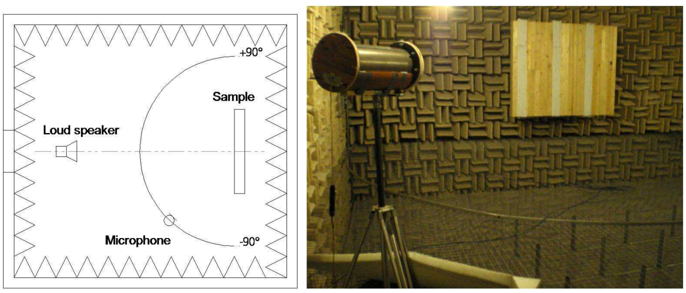

Figure 8: Measurement set-up to establish scatter diagrams (left). Example of the measuring set-up at a sound incidence angle of 30° (right)

Measurement is performed by means of the impulse response method. The measured impulse response consists of direct sound and reflections (cf. Figure 9).

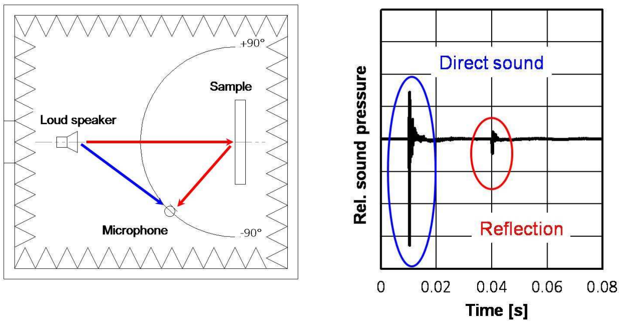

Figure 9: Incident and reflected sound wave (left), measured impulse response (right)

The reflection is separated in the time domain and then transformed into the frequency domain. The energy of the scattered waves was set proportional to the geometric reflection.

Figure 10 shows two examples of scatter diagrams for a structure with Λ_x=0.15 m. It is evident that the structure does not scatter at 630 Hz, as the incident wave length is greater than the period of the structure. Therefore, only geometrical reflection occurs.

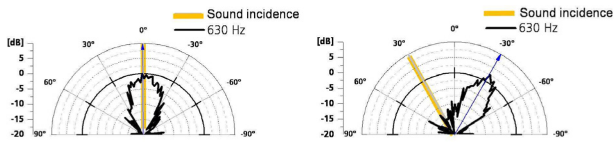

Figure 10: Examples of scatter diagrams Λ_x=0.15 m; normal (left) and oblique (30°) sound incidence.

Space harmonics, however, occur symmetrically around the geometric reflection in case of a structure of a period length of 0.80 m at a sound incidence angle of 0° (cf. Figure 11). These comply with the calculated ones. Space harmonics also occur at oblique sound incidence (30°). These are reflected backwards towards the sound source. Thus, the theoretically determined values are verified (cf. Figure 11).

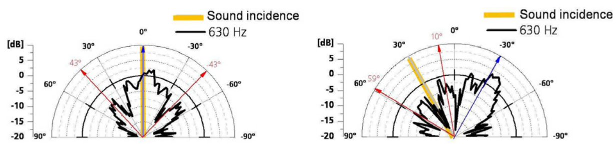

Figure 11: Examples of scatter diagrams at a period length of 0.8 m and a sound incidence angle of 0° (left) or 30° (right). Calculated angles of the geometric reflection (blue arrow) and the space harmonics (red arrows).

Summary

To characterize plane, periodic sound absorbers, different measurement methods can be applied for directional, random and normal sound incidence.

The scattering coefficient method allows a very good separation of the geometric reflection from the scattered ratio at random sound incidence. An absolute value is determined quantizing the structure with regard to its scatter behaviour.
Scattering diagrams also offer the possibility to separate the geometric reflection from space harmonics at directional sound incidence. Results determined experimentally are in good agreementwith the calculations. However, this method only allows the determination of relative values. The measurement methods presented do not allow the separate consideration of the near and far field. Both methods require extensive measurements and complex evaluations.

References

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