Calculation of Head Related Transfer Functions of bats using the Boundary Element Method

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Introduction

The project ChiRoPing (Chiroptera, Robots, and Sonar) is carried out under the Seventh Framework Programme ICT Challenge 2: "Cognitive Systems, Interaction, Robotics". The project involves four universities: The University of Antwerp, the University of Edinburgh, the University of Ulm and the University of Southern Denmark, which work in a cross-disciplinary environment. A description of the project and of each of the partners can be found in the webpage of the project [1].

The project seeks to obtain knowledge about the effect of the geometric features of the bat's ears and head in relation to its echolocation abilities; and to make use of this knowledge to design robots with equivalent artificial senses. Four species of bats are selected for the study and detailed models of their heads including ears, mouth and nose are obtained through CT scans. However, the present study is focussed on one of the species, the Myotis daubentonii.

Since, the bat operates at high frequencies (20 to more than 120 kHz) and as the geometry of its ears is very complicated and involves thin surfaces, the numerical modelling is challenging. The paper reports preliminary results of the study and investigates some possibilities of reducing the computational complexity of the models.

Obtaining the numerical model

The numerical model of the Myotis daubentonii is a discretized triangular mesh as shown in Figure 1. The triangular mesh is obtained though a micro-CT scan of the bat and subsequent data analysis and reduction at the University of Antwerp. For the initial calculations a mesh size corresponding to a maximum element length of 1 mm was chosen, which leads to an upper frequency limit of about 70 kHz using a rule of thumb of about 5 linear elements per wavelength. The resulting mesh has 5714 elements and 2847 nodes.

The Boundary Element Method

For the numerical computations an open source direct collocation Boundary Element Method (BEM) is applied [2]. The BEM is based on a discretized version of Helmholtz integral equation:

$$C(P)p(P) = \int_S p(Q)\frac{\partial G(P,Q)}{\partial n}dS + \int_S v_n(Q)G(P,Q)dS + p_{inc}(P), \qquad (1)$$

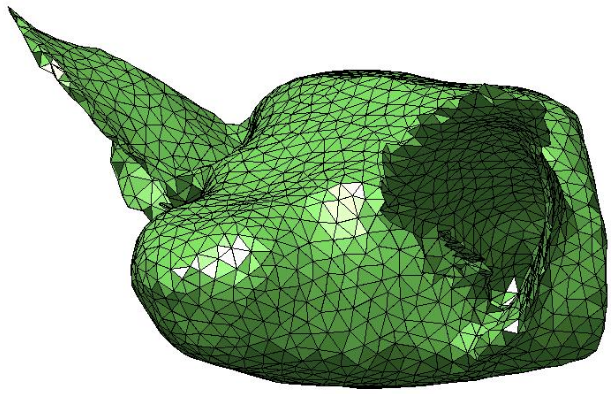

Figure 1: The model of Myotis daubentonii, ready for computations. The geometry is discretized into 5714 linear triangular elements and 2847 nodes.

which relates the sound pressure at a point P to the integral of the sound pressure $p(Q)$ and the normal particle velocity $v_n(Q)$ over a closed surface S. $G(P,Q)$ is the free-space Green's function between points P and Q; and $p_{inc}(P)$ is the undisturbed sound pressure of an (eventual) incoming sound wave. $C(P)$ are geometrical constants and equals the objects solid angle at P taken from the computational domain. Through discretization the integral equation is approximated by a matrix equation:

$$\mathbf{0} = (\mathbf{A} - \mathbf{C})\mathbf{p} + \mathbf{B}\mathbf{v} + \mathbf{p}_{\mathbf{inc}}, \qquad (2)$$

in which the matrix elements a_{mn} and b_{mn} of matrices **A** and **B** respectively are integrals over the triangular elements:

$$a_{mn} = \int_\Delta -N_\alpha(1+jkR)\frac{e^{-jkR}}{R^2}\frac{\partial R}{\partial n}J(x_1,x_2)dS \qquad (3)$$

and

$$b_{mn} = \int_\Delta N_\alpha \frac{e^{-jkR}}{R}J(x_1,x_2)dS \qquad (4)$$

Here N_α (α=1,2 and 3) are the linear shape functions and $J(x_1,x_2)$ is the Jacobian of the transformation to the local coordinate system of the triangular parent element.

In this study the head of the bat is assumed to be rigid, so the **v** term of equation (2) is zero, and the excitation of the model is a plane wave incident from different angles. However, in order to check the numerical computations, the so-called One Point Source test (OPS test) was made. In the OPS test a point source is created, usually inside the body. The free space velocity from this source is calculated at the

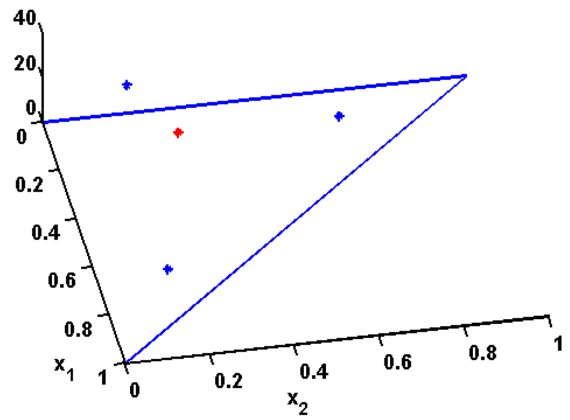

Figure 2: A near-singular integrand improperly resolved with the standard 3 point integration formula. The red dot is the projection of the collocation point and the blue dots are the integration points.

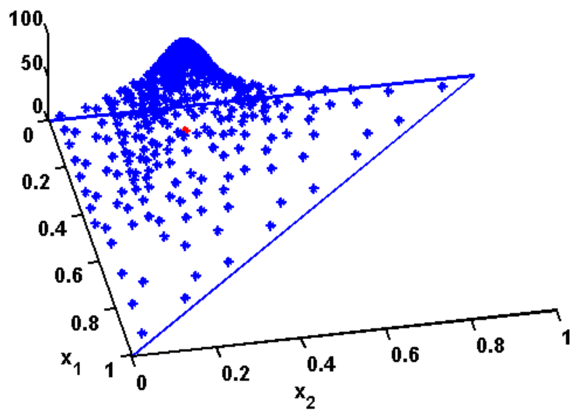

Figure 3: A near-singular integrand properly resolved with the recursive integration formula. The red dot is the projection of the collocation point and the blue dots are the integration points.

node positions, as if the model was “transparent”. This velocity’s normal component to the surface is taken as an input to calculate numerically the sound pressure over the boundary. The test is passed if this surface pressure is equal to that of the undisturbed monopole. The OPS test revealed characteristic frequencies in the higher frequency range, which were dealt with using a few CHIEF points [3].

The thin shape breakdown

It is well known that thin shapes and narrow gaps can lead to erroneous solutions in the resulting near-singular behaviour of the integrands in equations (3) and (4) [4]. Here the strategy of subdivision of the integrand as suggested in reference [5] is used in a version for triangular elements. The implementation involves a recursive algorithm in which subdivision of the triangles is carried until the near-singular integral is appropriately resolved. The stopping criterion of the algorithm is based on the distance to the collocation point P, the relative size of the present subdivided element and its distance to the projection of the collocation point onto the element plane. Figure 2 shows the inadequate resolution of a near singular integrand using the standard three integration points in each element, whereas figure 3 shows the same integrand properly resolved through the clustering of integration points by the recursive algorithm.

Frequency interpolation

The bats echolocate by emitting broard band sounds in a wide frequency range. Therefore, it is desired that the numerical calculations can be carried out with high frequency resolution in a wide frequency range.

Since the matrices in equation (2) are frequency dependant, a standard BEM implementation would require a large number of integrations for each frequency, leading to substantial time consumption for the process of setting up the system of equations (2).

For the computational model at hand, the set-up time of the matrices was about 1300 seconds (20 minutes) per frequency, whereas the solving of the set of equations took only about 10 seconds. The computer used was equipped with a 3.0 GHz Xeon processor and 16 Gb RAM running 64 bit SUSE Linux and Matlab R2009a.

In order to make the process faster, frequency interpolation has been used. The idea is to calculate explicitly the matrices A and B only at a few frequencies and then to interpolate the matrix elements in equations (3) and (4). Such a scheme could for linear interpolation be:

$$a_{mn}(k) \approx \frac{k_1 - k}{k_1 - k_0} a_{mn}(k_0) + \frac{k - k_0}{k_1 - k_0} a_{mn}(k_1) \qquad (5)$$

in which k_0 and k_1 denote the two wavenumbers (frequencies) where matrices are calculated explicitly and k is the wavenumber for which the interpolation is performed ($k_0<k<k_1$). A similar interpolation could be carried out for b_{mn}.

However, the exponential e^{-jkR} in equations (3) and (4) varies very fast with k for large R’s, which leads to large interpolation errors. Instead, a generalized version of the procedure suggested in reference [6] was used.

Since each element is defined to be relatively small compared to the wavelength (about one-fifth of a wavelength at the highest frequency), a slow variation with respect to the wavenumber may be obtained by scaling each matrix element with a factor of $e^{jkR_{mn}}$ – see figure 4. Here R_{mn} denotes the distance between the collocation point P (row m in matrices **A** and **B**) and the node relating to the element being integrated with the corresponding shape function N_α (column n in **A** and **B**). Hence scaled versions of equations (3) and (4) are found as:

$$\hat{a}_{mn} = \int_\Delta - N_\alpha (1 + jkR) \frac{e^{-jk(R - R_{mn})}}{R^2} \frac{\partial R}{\partial n} J(x_1, x_2) dS \qquad (6)$$

and

$$\widehat{b}_{mn} = \int_{\Delta} N_{\alpha} \frac{e^{-jk(R-R_{mn})}}{R} J(x_1, x_2)dS \tag{7}$$

respectively. Since R-R_{mn} is less than the element size $k(R-R_{mn})$ will be less than $2\pi/5$ if five elements per wavelength are used, and typically much smaller.

Hence, the interpolation is carried out on the scaled matrix elements, a quadratic version becomes:

$$\widehat{a}_{mn}(k) \approx \frac{k_1 - k}{k_1 - k_0}\frac{k_2 - k}{k_2 - k_0}\widehat{a}_{mn}(k_0) + \frac{k - k_0}{k_1 - k_0}\frac{k_2 - k}{k_2 - k_1}\widehat{a}_{mn}(k_1) + \frac{k - k_0}{k_2 - k_0}\frac{k - k_1}{k_2 - k_1}\widehat{a}_{mn}(k_2) \tag{8}$$

One drawback of the interpolation scheme is the need of saving the real and symmetric matrix **R** containing the R_{mn} – at least if a quick interpolation is desired. For the present case each of the 2847 by 2847 complex **A** matrices took up about 130 Mb of RAM, and it was possible to have all matrices in the RAM at the same time. Therefore, a quadratic interpolation could be performed in a little more than one second, which is much less that the 1300 seconds needed to set-up the system of equations explicitly.

Results

A few simulations have been made. The quantity of interest is the Head Related Transfer Function (HRTF) of the bat, which is defined as the sound pressure at the ear-drum of the bat, when the head is exposed to a plane wave coming from a specified direction, relative to the undisturbed plane wave at a reference position – normally in the centre of the head. Hence, the HRTF is a function of solid angle and frequency for each animal.

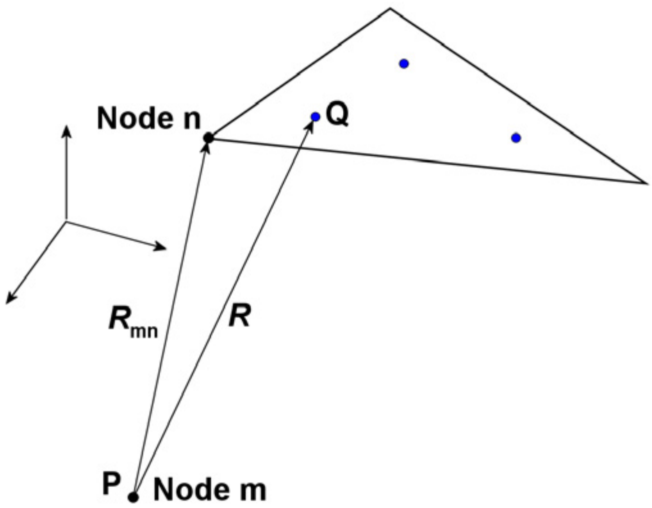

Figure 4: Scaling the matrix elements. The distance R is almost equal to R_{mn}

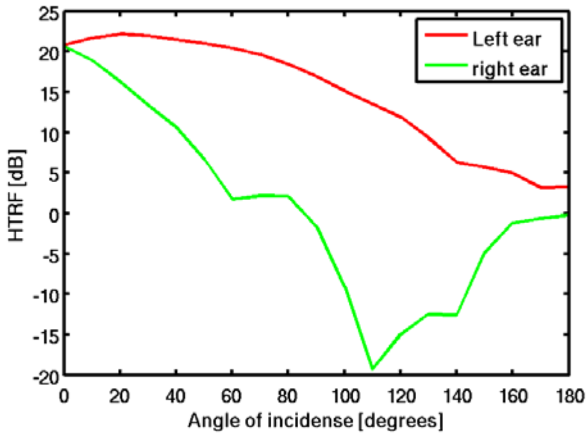

Figure 5: HRTF at f=40 kHz for horizontal incidence as a function of horizontal angle.

Figure 5 shows the HRTF of the Myotis daubentonii for a frequency of 40 kHz as a function of incidence angle for the horizontal plane, so that zero degrees denoted frontal incidence and 90 degrees corresponds to incidence directly from the left. It is evident that the level difference becomes very large even for relatively small incidence angles, which could help the bat in determining the direction to the sound source with high accuracy.

Figure 6 shows the HRTF as a function of frequency for zero degree incidence (frontal incidence). It is seen that the response of the left and of the right ear are similar, but not identical, which is in agreement with the fact that the bat is more or less symmetric in a vertical plane though the centre of its head, but not in perfect symmetry.

The responses in figure 6 have been calculated using the quadratic frequency interpolation method – equation (8) in between three frequencies: 20, 40 and 60 kHz. At 30 and 50 kHz control calculations have been carried out by explicitly calculating the coefficient matrices at these frequencies. The results of these calculations are denoted by stars in the plot, and confirm the accuracy of the interpolation scheme.

Finally, figure 7 shows the HRTF for horizontal incidence at 30 degrees to the left as a function of frequency. The figure shows that the level difference between the two ears becomes very large at high frequencies. Again quadratic frequency interpolation has been carried out, and it is worth noting that the interpolation agrees very well with the control calculations (denoted with stars), despite the large variation of the HRTF at the right ear in the range from 40 to 60 kHz.

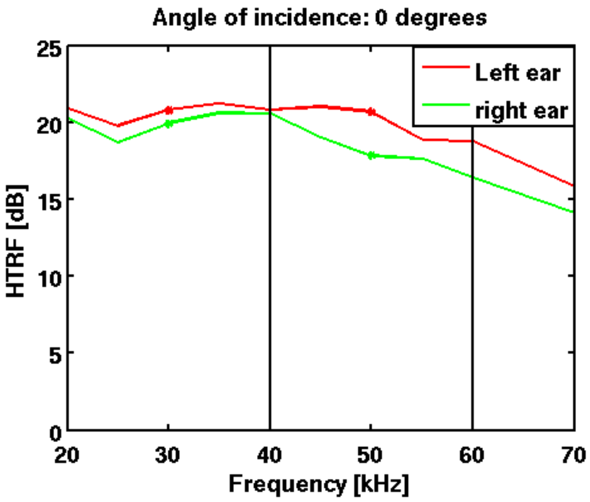

Figure 6: HRTF as a function of frequency for frontal incidence using frequency interpolation. Differences between the two curves are due to the fact that the head of the specimen of the Myotis daubentonii is not perfectly symmetric.

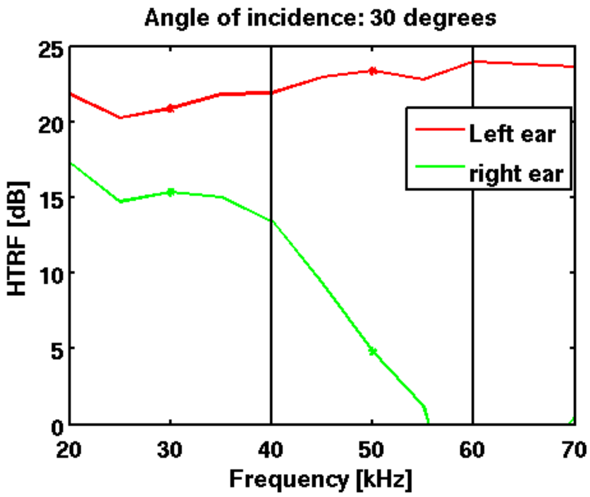

Figure 7: HRTF as a function of frequency at 30 degrees incidence to the left using frequency interpolation.

Discussion

The initial mesh used in this paper may be used for relatively low frequencies. For high frequencies, a finer mesh is needed. Table 1 summarizes the computational characteristics of the rough model described in the present paper, and estimates the characteristics of the high-frequency model.

	Rough model f_{max}=70 kHz	Fine model f_{max}=120 kHz (estimated)
Number of nodes/elements	2847/5714	7255/14534
Time for setting up the equations (s)	1300	8500
Time for solving the system of equations (s)	10	120
Time for interpolation (s)	1	<10
Matrix size (Mb)	130	850

Table 1: Computational characteristics of the models

Conclusions

Head Related Transfer Functions for the Myotis daubentonii have been estimated using a Boundary Element Method. Special care has been taken with regard to the thin shape of the ears, and frequency interpolation has been employed in order to speed up the calculations.

Acknowledgements

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