Frequency domain simulation of a Scottish bagpipe chanter with different tapers using the Harmonic Balance Method

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Introduction

Bagpipe makers have experimented with the bore of their chanters over the years by trial and error, in order to arrive to their current design. Therefore, they are usually reluctant to give detailed information about how their chanters are made. A quick survey among Scottish bagpipe makers revealed that the chanter bore profiles of Scottish bagpipes (both Great Highland bagpipe and Border bagpipe) differ significantly between them. However, they have the same broad configuration: 1. Reed seat, which is where the reed and staple are inserted, 2. throat and 3. main bore (see Figure 1). Each bagpipe maker usually makes reeds that fit only his chanter, that is specifically matched to the instrument he designed.

The main bore, for both Great Highland and Border bagpipes, is conical, the taper of which varies between 1.5 and 4 degrees. An important difference between Great Highland and Border bagpipes is that the latter has a narrower taper than the former, since it is designed to be played indoors. Additionally, the Border bagpipe can be played either with bellows, or mouth blown like the Great Highland bagpipe.

Bagpipe makers believe that there are three crucial parameters that affect the behaviour of the chanter: 1. the diameter and length of the throat (which is usually cylindrical, but can also be conical), 2. the position of where the cones meet, and 3. the individual tapers of the cones that form the main bore. Bagpipe makers tend to keep these details to themselves.

This study takes a simplified case, where the chanter is assumed to have only two tapers, and the two cones meet at the middle of the instrument. The effect of having two tapers instead of only one single cone is studied, as well as the effect of increasing this bottom taper, by means of computer simulation of the reed and air column system.

The simulation solves a simple model with the Harmonic Balance method, resulting in the spectrum of the oscillating pressure inside the mouthpiece p, from which plots of RMS and playing frequency were made. In order to study the effect of the different bore configuration on the radiated sound of the instrument, it is necessary to multiply the spectrum of p times the transfer function of the bore. Once the spectrum of the radiated sound was obtained, the spectral centroid (which is regarded as being a measure of "brightness" of the sound, see for example [6], [8]) was also calculated and plotted.

In the following sections, the bore configurations of the virtual chanters are described. The physical model used to simulate the pressure inside the mouthpiece p is presented, followed by the results obtained for each bore configuration, as well as a discussion on the comparison of the radiated sound obtained for all instruments.

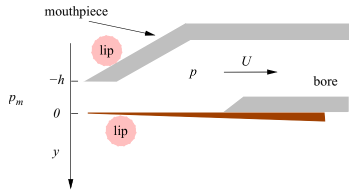

Figure 2: Schematic of a clarinet mouthpiece

Virtual Chanters

A schematic of the bore profile of the virtual instruments that were studied here is shown in Figure 1. It has five main parts: 1. The reed, 2. the staple, 3. the reed seat, 4. the throat, and 5. the main bore. The reed used for the virtual chanter was a cylinder which has approximately the same volume as that of a Border bagpipe reed. Pipers usually change the intonation of the chanter by inserting or pulling out the reed into or out of the reed seat. This is why it was chosen to leave a small portion of the reed seat.

Physical Model

The chanter and reed system are modelled as a self-sustained oscillator with a linear exciter (the reed) that is coupled non linearly to a linear resonator (the air column). This section presents the equations that were used to model these three components.

Reed

Almeida, et al. [1] have provided evidence that the two blades of a double reed have symmetric displacement. This means that the motion of only one blade needs to be modelled as a simple harmonic oscillator:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + g_r \frac{\mathrm{d}y}{\mathrm{d}t} + \omega_r^2 y = -\frac{1}{\mu_r} \Delta P \qquad (1)$$

where y is the displacement of the reed, g_r its damping factor, ω_r its resonance frequency, μ_r its mass per unit area, $\Delta P = p_m - p$, p_m is the pressure inside the mouth or wind cap and p is the pressure inside the reed. The stiffness k of the reed is

$$k = \mu_r \omega_r^2 \qquad (2)$$

This linear approximation only holds for non-beating reeds [4], [5].

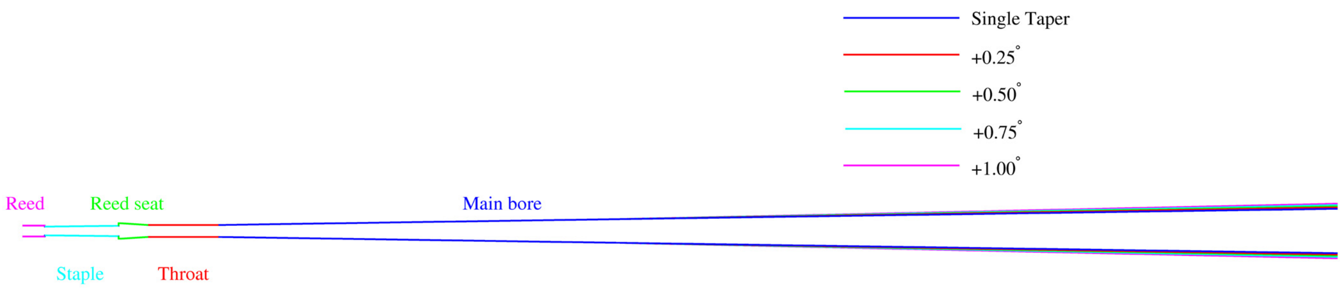

Figure 1: Schematic showing the bore profile of the five virtual instruments studied

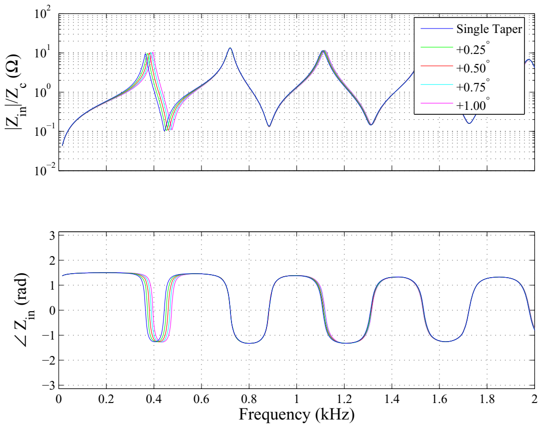

Figure 3: Calculated input impedance of the chanter

The maximum negative value that y can take is $-h$ (see Figure 2), at which point the reed gap is closed and the air flow into the mouthpiece is completely blocked. This occurs when the mouth pressure p_m is equal or greater to the closing pressure p_M:

$$p_M = \mu_r \omega_r^2 h = kh \tag{3}$$

Air column

The air column is usually characterised by its input impedance Z_{in}, which describes the interaction between the volume flow and the pressure inside the mouthpiece. in the frequency domain:

$$P(\omega) = Z_{in}(\omega)U(\omega) \tag{4}$$

Z_{in} was calculated using the "Simulate Air Column" feature of the program VIAS [9], by specifying the bore profile of the chanter (see previous section). It is shown in Figure 3.

Nonlinear coupling

The volume flow is related to the pressure across the mouthpiece by following Bernoulli's equation as follows [7]:

$$U = w(y+h)\sqrt{\frac{2\Delta P}{\rho}} sign(\Delta P), \tag{5}$$

where w is the width of the reed channel, ρ is the density of air, and $\Delta P = p_m - p$ is the pressure difference across the reed.

Harmonic Balance Method

Equations 1 (reed), 4 (air column) and 5 (nonlinear coupling) can be solved by the Harmonic Balance Method [4]. To keep these equations as general as possible, these equations are converted into dimensionless quantities by substituting:

$$\tilde{y} = \frac{y}{h} \tag{6}$$

$$\tilde{p} = \frac{p}{p_M} \tag{7}$$

$$\tilde{t} = t\omega_p \tag{8}$$

$$\gamma = \frac{p_m}{p_M} \tag{9}$$

where ω_p is the angular frequency of the first resonance peak of the air column. Similarly, equation 1 using dimensionless quantities becomes:

$$M\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}\tilde{t}^2} + R\frac{\mathrm{d}\tilde{y}}{\mathrm{d}\tilde{t}} + K\tilde{y} = \tilde{p} - \gamma \tag{10}$$

The parameters:

$$M = \left(\frac{\omega_p}{\omega_r}\right)^2 \tag{11}$$

$$R = \frac{\omega_p g_r}{\omega_r^2} \tag{12}$$

are the dimensionless mass and damping respectively, and since the reed closes when $p_m = p_M$, $K = 1$ (dimensionless stiffness) [4].

Similarly, equation 5 becomes:

$$\tilde{U}(\tilde{p},\tilde{y}) = \zeta(1+\tilde{y})\sqrt{|\gamma - \tilde{p}|} sign(\gamma - \tilde{p}) \tag{13}$$

as long as $\tilde{y} > -1$, otherwise $\tilde{U}(\tilde{p},\tilde{y}) = 0$ [4]. The "embouchure" parameter

$$\zeta = Z_0 wh\sqrt{\frac{2}{\rho p_M}} \tag{14}$$

is a parameter that characterises the mouthpiece [4].

Finally, the dimensionless form of the input impedance is obtained by:

$$\tilde{Z}_{in} = \frac{Z_{in}}{Z_0} \tag{15}$$

$$Z_0 = \frac{\rho c}{S} \tag{16}$$

S being the crosssectional area of the air column (cylindrical section) at the reed input. The dimensionless quantities in the frequency domain are:

$$\tilde{P}(\omega) = \tilde{Z}_{in}(\omega)\tilde{U}(\omega) \tag{17}$$

p_{th} [kPa]	$\omega_r/2\pi$ [kHz]	g_r [Hz]	H [mm]
4	4.5	300	0.25

Table 1: Reed parameters obtained experimentally [2]

The program harmbal [10] requires the parameters R, M and ζ, which are specified in a parameter file. In the version of harmbal used in this study (v2.0), the input impedance of the air column can be specified in a file. For a detailed discussion on the Harmonic Balance Method, as well as how the program harmbal solves this model, the reader is referred to [5], [4] and [3].

Physical parameters of the reed

The parameters of the reed that were used in the model were chosen based on the results presented in [2]. It is assumed that $p_M = 3p_{th}$, where p_{th} is the threshold pressure, that is, the minimum pressure required to start the vibrations of the instrument. These parameters are shown in Table 1.

Reed parameter calculation

The dimensionless parameters R, M and ζ are calculated with equations 11, 12 and 14, using the parameters shown in Table 1. The speed of sound was taken to be $c = 343.57\frac{\mathrm{m}}{\mathrm{s}}$, and the density of air $\rho = 1.19\frac{\mathrm{kg}}{\mathrm{m}^3}$, which are the values VIAS uses to do the input impedance simulation at the default temperature (21°C). Since the reed parameters were constant for all bore configurations, as well as the crosssectional area at the input of the instrument (and hence Z_0), the parameter ζ was the same for all configurations: $\zeta = 1.227$. The parameters M and R depend on both ω_r (constant) and ω_p, the latter being dependant on each particular configuration. The actual parameters that were passed to harmbal, are presented in Table 2.

Bore	**Parameter**		
	$\omega_p/2\pi$ (Hz)	M ($\times10^{-3}$)	R ($\times10^{-3}$)
Single	364.15	6.5484	5.3948
+0.25°	373.14	6.8758	5.5280
+0.50°	380.63	7.1547	5.6390
+0.75°	388.13	7.4391	5.7500
+1.00°	395.62	7.7291	5.8610

Table 2: Reed parameters used for the model

Results

Once the spectrum inside the mouthpiece p was obtained, the RMS amplitude and the pitch $\mathcal{P}$ in cents relative to the frequency of the note $F_4^{\#}$ were calculated as follows:

$$RMS = \sqrt{\sum_{k=1}^{N} A_k^2} \tag{18}$$

$$\mathcal{P} = 1200 \cdot log_2\left(\frac{f_0}{370}\right) \tag{19}$$

where k is the harmonic number, A_k is the amplitude of the k^{th} harmonic, N is the total number of calculated harmonics (20), and f_0 is the fundamental frequency of the sound.

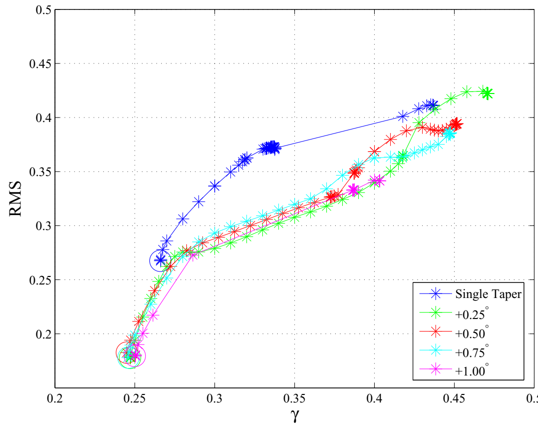

Figure 4: RMS amplitude of p (pressure inside the mouthpiece as solved by harmbal) vs γ for all five bore configurations

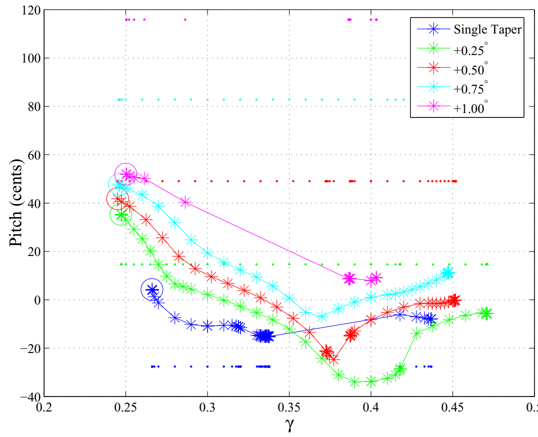

Figure 5: Pitch vs. γ for all five bore configurations. 0 cents corresponds to the frequency of the note $F_4^{\#}$ (370 Hz). The horizontal dotted lines represent frequencies of the first air column resonance.

Figure 4 shows a plot of RMS vs γ for all five configurations. The bore configurations that consisted of two tapers had essentially the same RMS curve. The configuration with the single taper had higher RMS values. Figure 5 shows a plot of pitch vs γ for all five configurations. Whenever the bottom taper is increased, the pitch also increases. The difference in pitch between single taper and the widest bottom taper can be as much as 50 cents.

The radiated sound spectrum was obtained from the spectrum of p, by multiplying the latter times the transfer function of the instrument (between the mouthpiece and the end of the chanter), which is shown in Figure 6. From the radiated sound spectrum, the spectral centroid SC was calculated as follows:

$$SC = f_0 \cdot \frac{\sum_{k=1}^{N} k \cdot A_k}{\sum_{k=1}^{N} A_k} \tag{20}$$

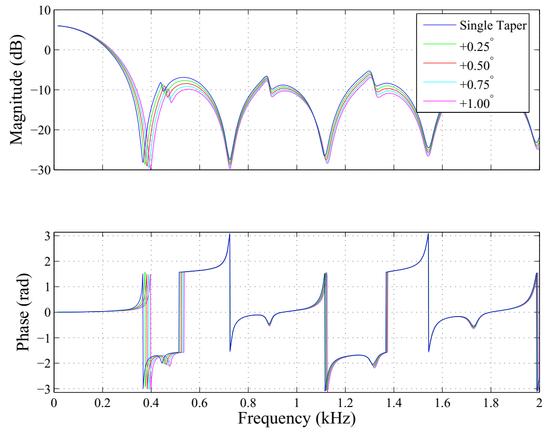

Figure 6: Calculated transfer function of the chanter for five bore configurations

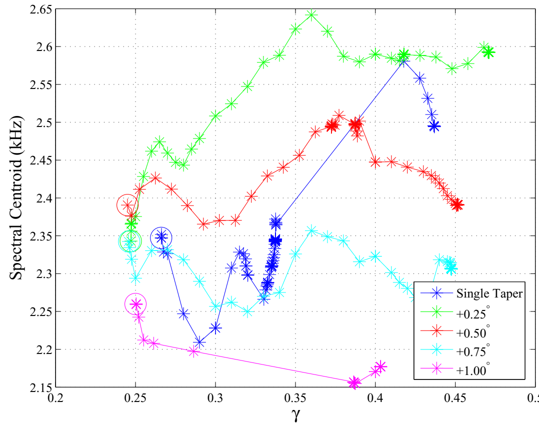

Figure 7: Spectral centroid vs. γ for all five bore configurations

Figure 7 shows a plot of SC vs γ. Starting from a single taper, increasing the bottom taper by 0.25° will result in a dramatic increase in spectral centroid, but increasing the taper even further will bring the spectral centroid back down again. If a bagpipe maker wished to modify a single tapered chanter in order to make it sound brighter, a slight increase would suffice, while a more dramatic increase will actually have the opposite effect.

Nature of the bifurcation

The left limit on each branch shown in Figure 4 corresponds to the minimum value of γ below which Harmbal cannot find a solution. As seen, these limit points correspond to RMS values still large. The question is: foreach branch, is there a portion for lower γ (for which Harmbal cannot converge) that would reach zero RMS values through a direct Hopf bifurcation? The answer is no, because it can be checked through a linear stability analysis (not detailed here) that the frequency of that solution branch (around $F_4^{\#}$), such as that shown in Figure 5, emerges from the non-oscillating solution for $\gamma_{th} = 0.37$. Therefore, in terms of bifurcation, it suggests that the limit left point on each branch is a turning point, and that the branches displayed do not emerge from a direct Hopf bifurcation. In terms of playability, it means that it is not possible to play these frequencies for the corresponding γ by simply increasing γ from zero, and that the oscillation threshold when increasing γ from zero does not match the extinction threshold when decreasing γ (hysteresis).

Conclusions

A set of five virtual chanters with different bore configurations were designed in order to study the effect that the bottom taper has on the sound of the instrument. A simple model of reed and air column system was used, and solved by the harmonic balance method. From the solutions of the simulation, the RMS, pitch and spectral centroid of the sound were calculated. The modification of the bottom taper proved to have a significant and monotonous influence on both pitch and RMS. This confirms what is experienced by makers: the angle of the bottom taper is a meaningful parameter that alters the characteristics of the sound played by a chanter.

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References

[1] André Almeida, Christophe Vergez, René Caussé, and Xavier Rodet. Physical study of double-reed instruments for application to sound-synthesis. In *Proceedings of the International Symposium on Musical Acoustics*, pages 215–220, Mexico City, Mexico, 2002.

[2] Sandra Carral and D. Murray Campbell. The influence of relative humidity on the physics and psychoacoustics of a Scottish bellows blown Border bagpipe chanter and reed. In *Proceedings of the Forum Acusticum Conference*, pages 379–384, Budapest, Hungary, 2005. Novotel Budapest Congress Centre.

[3] Snorre Farner, Christophe Vergez, Jean Kergomard, and Aude Lizée. Contribution to harmonic balance calculations of self-sustained periodic oscillations with focus on single-reed instruments. *Journal of the Acoustical Society of America*, 119(3):1794–1804, March 2006.

[4] Claudia Fritz, Snorre Farner, and Jean Kergomard. Some aspects of the harmonic balance method applied to the clarinet. *Applied Acoustics*, 65:1155–1180, 2004.

[5] J. Gilbert, J. Kergomard, and E. Ngoya. Calculation of the steady-state oscillations of a clarinet using the harmonic balance technique. *Journal of the Acoustical Society of America*, 86(1):35–41, 1989.

[6] R. A. Kendall and E. C. Carterette. Difference threshold for timbre related to spectral centroid. In *Proceedings of the 4^{th} International Conference on Music Perception and Cognition*, pages 97–101, Montreal, Canada, 1996.

[7] J. Kergomard. Elementary considerations on reed-instrument oscillations. In A. Hirschberg, J. Kergomard, and G. Weinreich, editors, *Mechanics of musical instruments*, Lecture notes CISM, chapter 6, pages 229–290. Springer, Vienna, 1996.

[8] Emery Schubert, Joe Wolfe, and Alex Tarnopolsky. Spectral centroid and timbre in complex, multiple instrumental textures. In *Proceedings of the 8^{th} International Conference on Music Perception and Cognition*, pages 654–657, Evanston, IL, USA, 2004.

[9] Reference to sofware VIAS. URL: `http://www.bias.at`

[10] Reference to sofware harmbal. URL: `http://www.pvv.ntnu.no/~farner/pub/harmbal`