# Energy resolution of complex vibro-acoustic problems

N. Totaro[1], J.L. Guyader[1]

[1] *INSA de Lyon – LVA, 25 bis avenue Jean Capelle, F-69621 Villeurbanne Cedex, Email:nicolas.totaro@insa-lyon.fr*

## Introduction

The present article deals with an energy resolution of complex vibro-acoustic problems. This method is based on SmEdA (Statistical modal Energy distribution Analysis) approach previously published for extending Statistical Energy Analysis (SEA) to systems where modal equipartition of energy is not achieved.

Modal energies are calculated from modal injected powers and modal coupling loss factors whatever the excitation even for systems with modal behavior. From the computing time point of view two basic advantages appears , the calculation is made directly for frequency averaged energies and modal coupling loss factors are simply extracted from modal characteristics of the uncoupled systems obtained with standard Finite Element method.

The approach provides also an analysis tool to detect the modal transmission path, very useful in early design stage. It is possible to easily identify couple of structural and acoustical modes most likely to transmit energy and estimate contributions of each structural mode to energy into the acoustical domain

One interesting point is the possibility of calculating energy maps into systems with very small increase of computation, additional assumption is necessary that will be discussed.

An application on a test case will be presented and compared with standard Finite Element method.

## SmEdA approach and modal energies

SmEdA [1,2,3] approach deals with modal energies of subsystems rather than global energies to overcome one of the most constraining assumption of SEA : energy equipartition.

SmEdA approach is based on basic equations that describe energy flow between two coupled oscillators. Using dual modal formulation stress/displacement and basic equations of SEA, the power flow $\Pi_{pq}$ between two coupled oscillators writes :

$$\Pi_{pq} = \beta_{pq}\left(E_p - E_q\right) \tag{1}$$

Where $E_p$ and $E_q$ are energies of oscillators $p$ and $q$ and $\beta_{pq}$ is the intermodal coupling factor between both modes.

SmEdA approach generalizes this equation to two coupled subsystems by considering each subsystem as a set of oscillators. In that case, a mode $p$ of subsystem 1 and a mode $q$ of subsystem 2 exchanges energy trough the intermodal coupling factor $\beta_{pq}^{12}$ which writes :

$$\beta_{pq}^{12} = \frac{\left(W_{pq}^{12}\right)^2}{M_p^1 M_q^2 \left(\omega_q^2\right)^2}\left(\frac{\eta_p^1\omega_p^1\left(\omega_q^2\right)^2 + \eta_q^2\omega_q^2\left(\omega_p^1\right)^2}{\left(\left(\omega_p^1\right)^2 - \left(\omega_q^2\right)^2\right)^2 + \left(\eta_p^1\omega_p^1 + \eta_q^2\omega_q^2\right)\left(\eta_p^1\omega_p^1\left(\omega_q^2\right)^2 + \eta_q^2\omega_q^2\left(\omega_p^1\right)^2\right)}\right) \tag{2}$$

Where $\eta_p^1$, $\omega_p^1$ and $M_p^1$ (resp. $\eta_q^2$, $\omega_p^1$ and $M_p^1$) are damping loss factors, the natural angular frequency and the modal mass of modes $p$ (resp. $q$) of subsystem 1 (resp. 2). The term $W_{pq}^{12}$ is the modalwork between displacement mode shape of mode p of subsystem 1 and stress mode shape of mode q of subsystem 2.

Finally, modal energies of coupled subsystems can be calculated solving the system of equations :

$$\frac{1}{\omega}\begin{Bmatrix}\left\{\Pi_p^1\right\}\\ \left\{\Pi_q^2\right\}\end{Bmatrix} = \begin{bmatrix}\left[\gamma_p^1\right] & \left[\beta_{pq}^{12}\right]\\ \left[\beta_{pq}^{12}\right] & \left[\gamma_q^2\right]\end{bmatrix}\begin{Bmatrix}\left\{E_p^1\right\}\\ \left\{E_q^2\right\}\end{Bmatrix} \tag{3}$$

Where $\left\{\Pi_p^1\right\}$ is a vector of modal power injected into modes of subsystem 1, $\left\{E_p^1\right\}$ is a vector of modal energies of modes of subsystem 1. $\left[\beta_{pq}^{12}\right]$ is a full matrix of intermodal coupling loss factors and the terms $\gamma_p^1$ of the diagonal matrix $\left[\gamma_p^1\right]$ are given by :

$$\gamma_p^1 = \eta_p^1\omega_p^1 + \sum_q \beta_{pq}^{12} \tag{4}$$

## Intermodal Coupling Factors

The intermodal coupling factor $\beta_{pq}^{12}$ can be considered as an analysis tool to identify couples of modes mainly responsible of energy transmission between subsystems. Indeed, as can be seen in equation 3, a high value of intermodal coupling factor is due to frequency coincidence and spatial coincidence between modes.

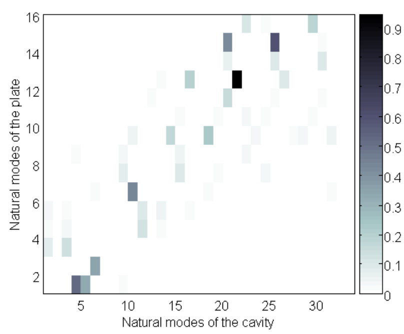


**Figure 1**: Intermodal coupling factors matrix between a rectangular plate and a cavity, frequency band [600…800]Hz.

Figure 1 shows a typical matrix of intermodal coupling factors between resonant modes of a plate and a cavity in the frequency band [600…800Hz]. This figure demonstrates that, in this frequency band, the mode (5,7) of the plate (natural frequency 749Hz) and the mode (4,0,2) of the cavity (natural frequency 746Hz) are mainly responsible of energy transmission between subsystems.

# Energy distribution into subsystems

SmEdA approach can be used to predict global energies into subsystems or to build a SEA model. It keeps same advantages as SEA : influence of damping and force location can be studied with simple and fast calculations.

In addition, even if SmEdA approach uses FEM results, it deals with modal bases of uncoupled subsystems which can be obtained up to higher frequency than for the whole system. The most time consuming step (calculation of modalworks between modes) has to be computed only once whatever the applied force or the modal damping. As a consequence, SmEda approach suits well to parametric study.

However, modal energies obtained with SmEdA approach can be used to predict an additional information : spatial energy distribution into subsystems.

The local energy at point M over a frequency band Δω can be written as the sum of intermodal contributions :

$$e^i(M,\Delta\omega) = \sum_r \sum_s e^i_{rs}(M,\Delta\omega) \quad (5)$$

Where $e^i_{rs}(M,\Delta\omega)$ is the intermodal contribution of modes *r* and *s* of subsystem *i*. In the following it will be supposed that sum of the off-diagonal terms ($r \neq s$) is negligible. This assumption fails for subsystems with modal overlap higher than unity.

Then, modal local energy $e^i_{rr}(M,\omega)$ is a function of modal amplitudes $a^i_r(\omega)$ and modal shapes $\phi^i_r(M)$ of the uncoupled subsystems :

$$e^i_{rr}(M,\Delta\omega) = \xi^i \left|a^i_r(\omega)\right|^2 \phi^{i\,2}_r(M) \quad (6)$$

Where $\xi^i$ is a constant depending on subsystem characteristics (structure or cavity). Integrating modal local energy over a frequency band Δω and over the space domain $D_i$, one writes :

$$\int_{D_i} e^i_{rr}(M,\Delta\omega)dM = E^i_r = \xi^i \left\langle \left|a^i_r(\omega)\right|^2 \right\rangle_{\Delta\omega} N^i_r \quad (7)$$

Where $N^i_r$ is the norm and $E^i_r$ is the modal energy of mode *r* of subsystem *i*. $E^i_r$ are modal energies handled by SmEdA approach. Finally, local energy at point M averaged over a frequency band Δω writes :

$$e^i(M,\Delta\omega) = \sum_r \frac{E^i_r}{N^i_r} \phi^{i\,2}_r(M) \quad (8)$$

As can be seen in equation (8), all terms of the sum are obtained during SmEda calculations. Thus no more time consuming calculations are needed to obtain an estimate of spatial energy distribution into subsystems.

Let's take the example of a rectangular simply supported plate coupled to a cavity as presented in figure 2.

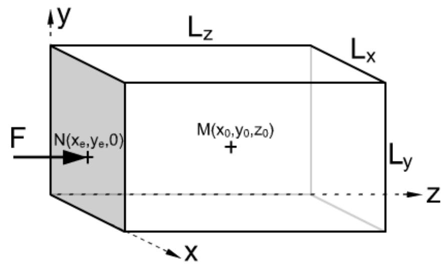


**Figure 2**: A vibrating rectangular plate radiates into a cavity with rigid walls.

The plate is excited by a random broad band point force. To apply SmEdA extension and predict energy maps into subsystems, modal bases of the uncoupled subsystems are calculated analytically. Then, SmEdA model is build and modal injected powers are calculated. Finally, modal energies of subsystems are obtained and used in equation (8) to estimate spatial energy distribution on the surface of the plate and two cutting planes in the cavity.

Figure 3 presents spatial energy distribution on these three planes in the frequency band [600…800]Hz. These results can be compared to those obtained with exact calculation presented in figure 4.

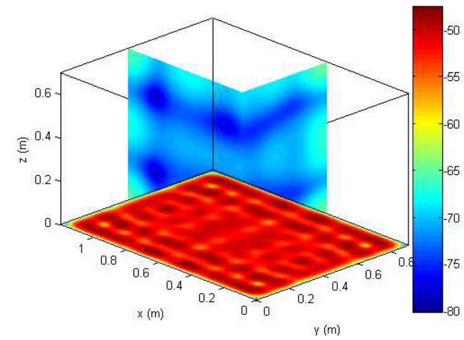


**Figure 3**: Spatial energy distribution in the frequency band [600…800]Hz on the plate and in the cavity obtained with SmEdA approach.

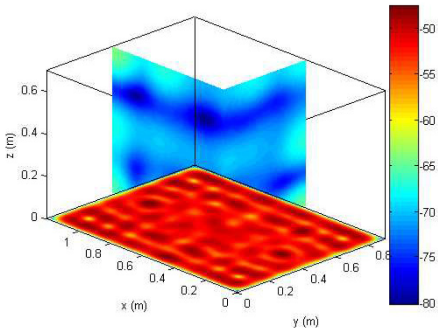


**Figure 4**: Spatial energy distribution in the frequency band [600…800]Hz on the plate and in the cavity obtained with exact calculation.

As can be seen in figures 3 and 4, SmEdA approach well predicts spatial energy distribution into subsystems. The main advantage of the SmEdA approach is the calculation time. When using standard pure tone response, one has to make the average over the frequency band to calculate energy whereas SmEdA gives directly modal energies in the frequency band and uses modal bases of uncoupled subsystems. In addition, the modification of subsystem damping and excitation is very easy in SmEdA because the modes shapes remain identical. From this point of view, the approach keeps the same advantage as standard SEA to analyze damping effect in power flow between subsystems.

Figure 5 shows the comparison between energy responses at one point inside the cavity calculated with exact calculation and SmEdA approach. It demonstrates that SmEdA approach is able to predict local energy in subsystems.

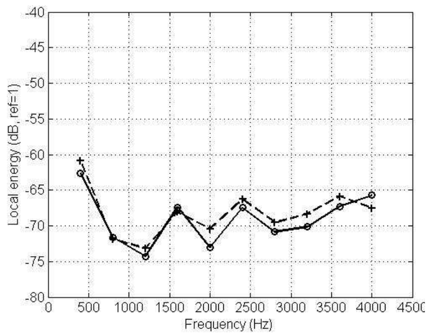


**Figure 5: energy response at one point inside the cavity. –o– exact calculation; --+-- : SmEda approach.**

# Conclusion

The SmEdA approach has been developed to extend classical SEA to subsystems with low modal overlap or for localized excitations. As a consequence, this method deals with modal energies instead of global energies as SEA. Usually, this method was used to finally predict energies in subsystems.

The present article proposed an extension of the SmEdA method to predict spatial energy distribution in subsystems. Thanks to a simplifying assumption, it is demonstrated that spatial energies distribution can be predicted using only modal energies and mode shapes of modes of the uncoupled subsystems. These quantities have already been calculated in SmEdA approach thus no more time consuming calculations are needed. This makes SmEdA extension faster than classical FEM simulation. Indeed, mode shapes of uncoupled subsystems remain identical whatever the damping or the source location. As a consequence, SmEdA suits well for a parametric study.

In addition, the intermodal coupling factors handled by SmEdA can be used as a tool to identify modes mainly responsible of energy transmission between subsystems. This can be an useful tool in early design stage of an industrial process.

# References

[1] L. MAXIT and J.-L. GUYADER. Estimation of SEA coupling loss factors using a dual formulation and FEM modal information, part I : Theory. Journal of Sound and Vibration, 239(5), p907-930, 2001.

[2] L. MAXIT and J.-L. GUYADER. Estimation of SEA coupling loss factors using a dual formulation and FEM modal information, part II : Numerical Applications. Journal of Sound and Vibration, 239(5), p931-948, 2001.

[3] N. TOTARO, C. DODARD, and J.L. GUYADER. SEA coupling loss factors of complex vibro-acoustic systems. Journal of Vibration and Acoustics, 131(2), 2009.