Nonlinearities in wind instruments at the example of organ pipes

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Introduction

We describe the reconstruction of mode-locked or synchronized dynamics of an organ pipe standing side-by-side to a loudspeaker. The speaker is varied in frequency and amplitude, or sound pressure level (SPL), respectively. This way, a detailed study of its effect on the organ pipe is possible. As a result the range of synchronization is found in the so-called Arnold Tongue, a triangular-shaped region in the above mentioned parameter space.

In order to construct an analytical model for this experiment, the pipe is modeled as an autonomous oscillator. In fact, it is rather the oscillating air sheet at the pipe mouth which is modeled, since this is the basic oscillating unit which eventually is the main source for the radiation of sound [1]. In the following, we concentrate on this model for the pipe and describe how it is constructed from data.

Synchronization of an organ pipe

The issue of synchronization, or mode-locking, has been discussed already several years ago [2, 3]. It is a quite general effect which goes along with the occurrence of nonlinear effects in musical instruments. Particularly important, mode locking influences the interaction of bow and string in bowed-string instruments due to the highly nonlinear stick-slip motion of the bow. However, basically all vibrations show nonlinearities once the amplitudes are high enough; of course this is to avoid usually, because then partials can turn to be no longer harmonic. There *is* a strongly nonlinear element in an organ pipe: exactly the air sheed which is a part of the jet produced at the pipe foot where the air is blown in from the wind supply [4, 5, 6]. That implies that there can be expected strong nonlinear effects when organ pipes are subject to other sound sources with high SPL.

In a recent publication the mutual influence of organ pipes has been described and quantitatively investigated; further, the effect of external driving by a loudspeaker has been considered. It has been shown that an autonomous oscillator model is sufficient to describe the effects of synchronization. In this contribution, we describe ongoing work on the analytical and numerical description of the oscillating unit in an organ pipe by numerical reconstruction of the underlying dynamical system.

Let us assume the description of an organ pipe by a two dimensional autonomous oscillator is correct. Then the corresponding equation reads

$$\ddot{x} + \omega_0^2 x + f(x, \dot{x}) = 0 \; , \qquad (1)$$

where x represents the amplitude of the oscillator, ω_0 is its natural frequency, and $f((x, \dot{x})$ is a nonlinear function accounting for energy input and dissipation. The external driving $g(t)$, in our case by a loudspeaker is modeled sinusoidally by a harmonic force with angular frequency $\omega_1 \simeq \omega_0$: $g(t) = \omega_1^2 R \cos(\omega_1 t)$. Now, two time scales are present in the system, a fast one $t_f = \frac{2\pi}{\omega_1} \simeq = \frac{2\pi}{\omega_0}$, and a slow one $t_s = \frac{2\pi}{\omega_1 - \omega_0}$, with $t_s \ll t_f$.

To investigat synchronization, one investigatesthe dependence on the slow time and averages over the fast one by using the ansatz $x(t) = A(t)\sin(\omega_1 t + \Psi)$, with $\Psi = \Phi - \omega_1 t$ the slow phase, being the difference of the phase of the oscillator, and external force. After a few steps [7], one obtains an equation for the phase difference:

$$\dot{\Psi} \;=\; -\Delta\omega + \epsilon F(\Psi) \; . \qquad (2)$$

The equation for the amplitude is slaved to the phase and is omitted here. The time-periodic function F contains the dependence on f and the driving function g. There are two parameters in this equation: the (angular) frequency difference $\Delta\omega$, and the coupling strength ϵ. The first is easily determined from measurements, the latter is related to the nonlinear admittance of the air-sheet and reflects the coupling of an incoming acoustical wave to the air-sheet.

Given this motivation, we can outline a program to find a realistic representation of the autonomous oscillator: 1) measure the full parameter space with respect to synchronization, 2) determine the equations for the oscillator from data, this implies especially a decent representation of the nonlinearity f, 3) Integrate the model equations found, 4) compare with the experimental results.

In the following pages, we show how to construct f from data for different working conditions of the organ pipe. The other results are reported elsewhere [8].

Construction of an oscillator model

The pipe we used for measurements [9] was wooden and closed at the upper end, tuned at 168.3 Hz. It was driven by an especially fabricated miniature organ [9] with a blower connected to the wind-belt and further by flexible tubes to the wind-chest. The wind pressure was 140±9 Pa, if not indicated otherwise - it turned out to be interesting to drive the pipe into an unstable regime for testing our reconstruction method; this is shown below, cf. Figure 1. Measurements took place inside a suitable anechoic box. The signal emitted by pipe was registered at 15 cm to either pipe and speaker. To ensure that the phase of the pipe is correctly detected, we carried out simultaneous control measurements inside and outside the pipe and at the microphone - all results were consistent. The sampling rate was $\Delta t = 5$ kHz and

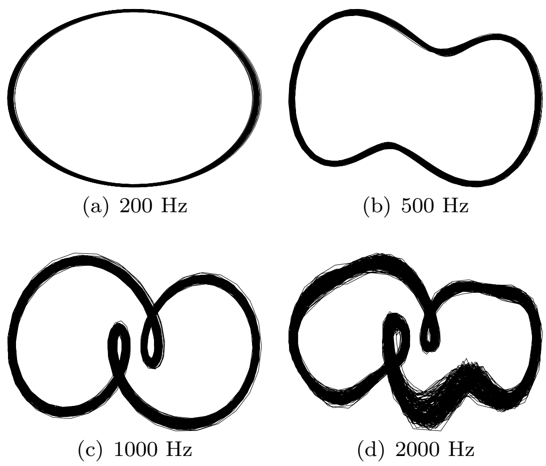

Figure 1: Embedding of the time series of the organ pipe with different cut-offs k_0. The wind pressure is 337 mBar such that the pipe showed signature of chaotic behaviour. The axes, x and y are omitted. With increasing cut-off nonlinearities play a bigger role. Of course for the first harmonic alone, a harmonic oscillation is found, cf. the top figure. So, one has to be careful to get rid of noise, but not to kill at the same time the nonlinear information hidden in the signal. Of course, a two-dimensional model cannot represent the trajectories for cut-off larger than 1000 Hz.

the time series consists of 60000 values.

We will apply the technique of differential embedding to a measured time series of an organ pipe. This means that from the measured signal $x(t)$ we will numerically calculate (estimate) the derivative $\dot{x}$ and then find the function $f(x, \dot{x})$ from a two-dimensional minimization procedure. The equations for the autonomous oscillator representation are rewritten in the form

$$\dot{x} = y \,, \quad (3)$$

$$\dot{y} = -\omega_0^2 x - f(x, y) \,. \quad (4)$$

The values for x and $\dot{x}$, and even $\ddot{x}$ are calculated for each sampling time t_n. We use spectral estimation [10], since this is perfectly suited for a periodic signal. To cope with noise, we FIR-filter the signal with a Butterworth filter of eigth order and varying cut-off frequency.

To find the function f, we minimize the functional

$$\chi^2 = \sum_n \left[y(\dot{t}_n) + \omega_0^2 x(t_n) + f\left(x(t_n), y(t_n)\right) \right]^2 \quad (5)$$

As a first step we visualize in Figure 1 the phase space of the oscillator by plotting x vs $\dot{x}$. To illustrate the effect of the filter at different cut-off frequencies we set the pipe to a very high wind supply of 337 mBar where it works in an unstable regime with intermittent chaos. However, the effect of nonlinearities is more expressed and so is the effect of cutting off higher harmonics.

Clearly, for a model of the chaotic vibrations, one has to step to a higher dimensional model. This is not subject of the considerations here, where we want to consider regular oscillations and a stable tone of the organ pipe.

For our signal at wind supply at 140 mBar we tested thoroughly for the best filter cut-off. It turned out that 1000 Hz were optimal, see Figure 3. With the good embedding we can enter the minimization (5) we want to model the function f. There is a huge variety of models, one can choose from. Best understood and consequently most appealing are additive models, because there, one can use the backfitting algorithm [11] which allows for a nonparametric fit of nonlinearities. Thus we write $f = f_1(x) + f_2(\dot{x})$. The result of the reconstruction is shown in Figure 2; both parts, f_1 and f_2 are highly nonlinear accounting for the existence of a limit cycle. The functions can now be approximated analytically or taken from the numerical representation in terms of cubic splines. Any way allows for the investigation of the determining dynamical properties - stability and frequency of the limit cycle. The stability of the limit cycle is evaluated numerically in integrating the equations with the numerically given function f. We find a repeller at zero and the limit cycle to be attracting as is necessary for a stable tone to exist.

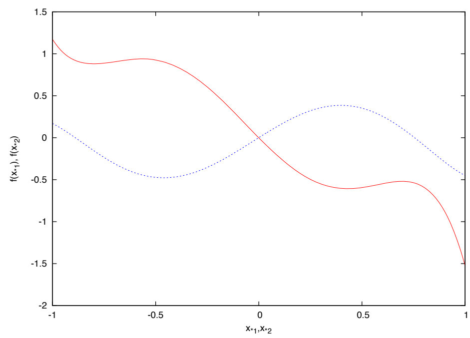

Figure 2: The function f is modeled additively by two functions $f_1(x)$ and $f_2(\dot{x})$. Both functions are nonlinear and allow for the existence of a limit cycle. The stability properties of the equations are confirmed numerically.

How good is this model in terms of musical quality? Of course the true organ sound is not yet recovered, however, if one compares not only the phase space reconstruction but as well the spectra of the system, a very good coincidence ist found, cf. Figure 3 Remarkably, not only the position of the harmonics is found quite nicely, but also the ratio of the amplitude of the harmonics is hit well. This is an indication that the coupling between the modes is recovered, since it depends on the type of nonlinearity.

Conclusion

We have presented a very direct way of modeling the tone of an organ pipe. The method is general and can be applied to any nonlinear autonomous (or not) system. Currently, the method is used for string instruments [12], but of course it may be used for digital sound generation in general. There are many ways to follow from this point. On one hand one can try to get better approximations for the system under consideration in order to find a perfect match to the organ pipe sound. Then, pipes of other dimensions should be investigated and compared. This way the sound of an organ could be numerically modeled quite efficiently compared with usual methods,

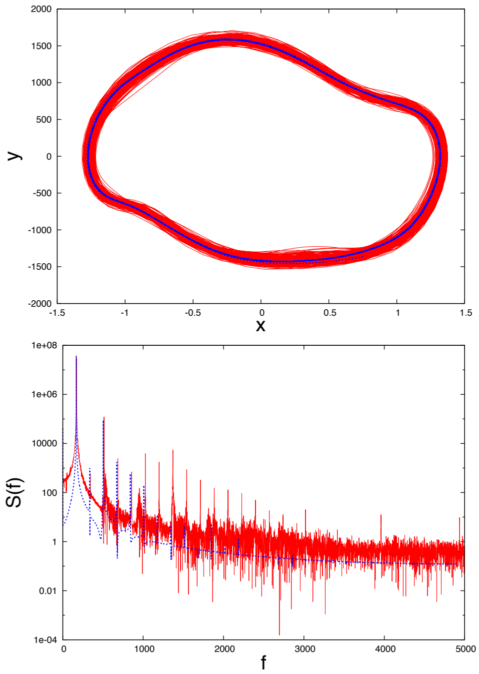

Figure 3: Two-dimensional phase space of the embedded signal. The wind pressure was 140±9 Pa, the cut-off frequency of the Butterworth filter was $1000Hz$. Top:The red curve corresponds to the measured signal with $y = dotx$ from numerical differentiation, the blue line results from the integration of an additive model for the nonlinearity f. Bottom: Power spectrum $S(f)$ to the time signal. The harmonics are precisely hit by our nonlinear model, and, even more important, the ratio of the amplitudes is very well recovered in the model.

where each pipe is modeled separately.Further, one can try to extend the results to higher dimensional systems, where chaos can occur and more complex phase-space patterns might appear. This task will be numerically much harder, but very appealing due to the complex sound which can be generated and the possibility to switch between chaos and order simply by changing a parameter of the system hidden in the nonlinear function f_1, and f_2. We want to hold this perspective open for the future.

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